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Abstract:

This document is intended for developers and architects who wish to design systems and applications that utilize threshold sharing schemes in an interoperable manner.

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Table of Contents

[1 Introduction 5](#_Toc85627897)

[1.1 Changes from earlier Versions 5](#_Toc85627898)

[1.2 Terminology 5](#_Toc85627899)

[2 Threshold Sharing Scheme 1 6](#_Toc85627900)

[2.1 Algorithms 6](#_Toc85627901)

[2.1.1 Field Representation and Mathematical Operations 6](#_Toc85627902)

[2.2 Usage 7](#_Toc85627903)

[2.2.1 Create Shares 7](#_Toc85627904)

[2.2.2 Recombine Shares 8](#_Toc85627905)

[2.3 Constants 9](#_Toc85627906)

[2.3.1 Polynomial 1 9](#_Toc85627907)

[2.3.1.1 LOG 9](#_Toc85627908)

[2.3.1.2 EXP 10](#_Toc85627909)

[2.3.2 Polynomial 2 11](#_Toc85627910)

[2.3.2.1 LOG 11](#_Toc85627911)

[2.3.2.2 EXP 12](#_Toc85627912)

[2.4 Test Vectors 13](#_Toc85627913)

[2.4.1 Polynomial 1 13](#_Toc85627914)

[2.4.1.1 Test Vector TV011B\_1 13](#_Toc85627915)

[2.4.1.2 Test Vector TV011B\_2 13](#_Toc85627916)

[2.4.1.3 Test Vector TV011B\_3 13](#_Toc85627917)

[2.4.1.4 Test Vector TV011B\_4 14](#_Toc85627918)

[2.4.1.5 Test Vector TV011B\_5 14](#_Toc85627919)

[2.4.1.6 Test Vector TV011B\_6 14](#_Toc85627920)

[2.4.2 Polynomial 2 15](#_Toc85627921)

[2.4.2.1 Test Vector TV011D\_1 15](#_Toc85627922)

[2.4.2.2 Test Vector TV011D\_2 15](#_Toc85627923)

[2.4.2.3 Test Vector TV011D\_3 15](#_Toc85627924)

[2.4.2.4 Test Vector TV011D\_4 15](#_Toc85627925)

[2.4.2.5 Test Vector TV011D\_5 16](#_Toc85627926)

[2.4.2.6 Test Vector TV011D\_6 16](#_Toc85627927)

[3 Conformance 17](#_Toc85627928)

[3.1 SAM Threshold Sharing Scheme 1 (TSS1) Implementation Conformance 17](#_Toc85627929)

[Appendix A. References 18](#_Toc85627930)

[A.1 Normative References 18](#_Toc85627931)

[A.2 Informative References 18](#_Toc85627932)

[Appendix B. Acknowledgments 19](#_Toc85627933)

[B.1 Participants 19](#_Toc85627934)

[Appendix C. Revision History 21](#_Toc85627935)

[Appendix D. Secret Sharing described in terms of matrix operations 22](#_Toc85627936)

[D.1 Create Shares 22](#_Toc85627937)

[D.1.1 Creating the P matrix 22](#_Toc85627938)

[D.1.2 Creating the S matrix 22](#_Toc85627939)

[D.1.3 Calculating the  matrix 23](#_Toc85627940)

[D.2 Recombine Shares 23](#_Toc85627941)

[Appendix E. Cryptol code for testing secret share creation and recombination 24](#_Toc85627942)

[E.1 Polynomial x8 + x4 + x3 + x + 1 24](#_Toc85627943)

[E.1.1 Table-based implementation 24](#_Toc85627944)

[E.1.2 “Native” GF(256)-based implementation 33](#_Toc85627945)

[E.2 Polynomial 41](#_Toc85627946)

[E.2.1 Table-based implementation 41](#_Toc85627947)

[E.2.2 “Native” GF(256)-based implementation 50](#_Toc85627948)

[Appendix F. Notices 58](#_Toc85627949)

# Introduction

[All text is normative unless otherwise labeled]

## Changes from earlier Versions

N/A

## Terminology

The key words "MUST", "MUST NOT", "REQUIRED", "SHALL", "SHALL NOT", "SHOULD", "SHOULD NOT", "RECOMMENDED", "MAY", and "OPTIONAL" in this document are to be interpreted as described in [[RFC2119](#RFC2119)] and [[RFC8174](#RFC8174)] when, and only when, they appear in all capitals, as shown here.

# Threshold Sharing Scheme 1

## Algorithms

Shamir’s secret sharing algorithm [[SHAMIR79](#SHAMIR79)] provides a mechanism to be able to:

[D]ivide data *D* into *n* pieces in such a way that *D* is easily reconstructable from any *k* pieces but even complete knowledge of *k-1* pieces reveals absolutely no information about *D*. This technique enables the construction of robust key management schemes for cryptographic systems that can function securely and reliably even when misfortunes destroy half the pieces and security breaches expose all but one of the remaining pieces.

Shamir’s algorithm is a threshold secret sharing scheme. When applied to protection of cryptographic keys, the pieces are referred to as either key shares or key splits of the original key. The scheme may be referred to as either *k*-of-*n* or *m*-of-*n* where *k* or *m* refers to the minimum number of shares or splits (the threshold) required to reassemble the original key and *n* refers to the total number of shares or splits.

Shamir’s scheme is based on polynomial interpolation. For interoperable systems the polynomial must be specified. The maximum number of shares or splits is determined by the specific polynomial.

In this specification, we select irreducible polynomials for interpolation over a finite field (Galois field) of the order 28 referred to as GF(28) or GF(256).

Within the finite field, we select between one of two polynomials (using the notation from [[FIPS197](#FIPS197)]):

1. {01} {1B} (283) which is x8 + x4 + x3 + x + 1

2. {01} {1D} (285) which is x8 + x4 + x3 + x2 + 1

The first polynomial (denoted throughout the rest of this document as 011B) is used within AES. The second polynomial (denoted throughout the rest of this document as 011D) is used within the RSA BSAFE range of SDKs from RSA Security and is included to enable compatibility with the broadest range of applications.

### Field Representation and Mathematical Operations

For efficient implementation we define (for each polynomial) two tables—one table represents the exponentiation function (EXP), while the other table provides a representation of the logarithmic function (LOG). The exponentiation function is also referred to as the antilog function (ALOG). The *i*-th element of each table is denoted as EXP[*i*] and LOG[*i*] respectively.

All bytes in the threshold secret sharing algorithm are interpreted as finite field elements on which addition, subtraction, multiplication and division are defined as follows:

1. The addition operation (GF\_ADD) takes two elements X and Y and returns the bitwise exclusive-or of its operands. Note that GF\_ADD(0, X) returns X.
2. The subtraction operation (GF\_SUB) is identical to addition, because the field has characteristic two.
3. The multiplication operation (GF\_MUL) takes two elements X and Y as input and, if X or Y is zero, returns zero; otherwise, the operation returns the value EXP[(LOG[X]+LOG[Y]) modulo 255].
4. The division operation (GF\_DIV) takes a dividend X and a divisor Y and computes X divided by Y. If X is zero, the operation returns zero. If Y is zero, an error condition is returned; otherwise, the operation returns the value EXP[(255-LOG[X]-LOG[Y]) modulo 255].

The exponentiation operation of raising X to the power of *i,* denoted by Xi, returns X multiplied by itself *i* times using GF\_MUL is defined as GF\_POW:

GF\_POW(X, i) = GF\_MUL(X, GF\_MUL(X, … GF\_MUL(X, X)))

The sum operation GF\_SUM(X, *n*) takes *n* elements from the vector X (i.e. X[0], X[1], …, X[*n*-1]) and adds them together using GF\_ADD operation:

GF\_SUM(X, *n*) = GF\_ADD(X[0], GF\_ADD(X[1], GF\_ADD(X[2], … GF\_ADD(X[*n*-2], X[*n*-1])))).

Likewise, the product operation (GF\_PROD) takes *n* elements from X and multiplies them together using the GF\_MUL operation:

GF\_PROD(X, *n*) = GF\_MUL(X[0], GF\_MUL(X[1], (GF\_MUL(X[2], … GF\_MUL(X[*n*-2], X[*n*-1])))))

For the specific values for the EXP and LOG tables for a given polynomial, refer to the Constants section of this specification.

## Usage

### Create Shares

The secret shares will be created from a secret, with length *L* bytes, where L ranges from zero to 65,534 (216 – 2) bytes. A threshold number of shares, *m*, will be required in order to recombine the secret. The total number of shares is designated by *n*, with the constraint that *m* ≤ *n* ≤ 255.

Individual shares are calculated by applying a function *F* (X, S) (see Appendix D) to a byte X (the share ID) and an *m*-byte input vector S:

 *F* (X, S) = GF\_SUM(Y[*i*]), where Y[*i*] = GF\_PROD(S[*i*], Xi) for *i* = 0, *m* – 1.

With input defined as:

* *n* = number of shares to be created.
* *m* = the threshold of shares required to be recombined to form the secret.
* *secret* = an *L*-byte vector containing the secret that will be split.
* *share\_id*[*i*]= a unique identifier (range 1 … 255) for share *i*, otherwise known as the share ID for share *i*.
* R = a vector of random bytes of length (*m* - 1) *L*.

And with output defined as:

* *share*[*i*]= a vector of shares (*i* = 0 … *n* – 1) with each share containing *L* + 1 bytes (note: the first byte contains *share\_id*[*i*]).

The pseudo code algorithm for calculating the *n* shares is:

**CheckErrors:**

 **if** *L* < 0 or *L* > 65,534 **then**

 throw an error and exit

 **endif**

 **if** *m* < 1 or *m* > 255 **then**

 throw an error and exit

 **endif**

 **if** *n* < *m* or *n* > 255 **then**

 throw an error and exit

 **endif**

 **for** *i* = 0 to *n* - 1

 **do**

 **if** share\_id[*i*] not unique **then**

 throw an error and exit

 **endif**

 **if** share\_id[*i*] == 0 **or** share\_id[*i*] > 255 **then**

 throw an error and exit

 **endif**

 **done**

**CalculateShares:**

 **for** *i* = 0 to *n* - 1

 **do**

 share[*i*][0] = share\_id[*i*]

 **done**

 counter = 0

 **for** *i* = 0 to *L* – 1

 **do**

 **for** *j* = 1 to *m* – 1

 **do**

S[*i*][0] = secret[*i*]

 S[*i*][*j*] = R[counter]

 counter = counter + 1

 **done**

 **done**

 **for** *i* = 0 to *n* – 1

 **do**

 **for** *j* = 0 to *L* - 1

 **do**

 share[*i*][*j* + 1] = *F*(shares[i][0], S[*j*])

 **done**

 **done**

**F(X, Sj):**

result = 0

 **for** *k* = 0 **to** *m* - 1

 **do**

result = GF\_ADD(result, GF\_MUL(Sj[*k*], GF\_POW(X,*k*)))

 **done**

 **return** result

### Recombine Shares

Recombining the shares requires *m* of the shares to be provided. If more than *m* shares are provided, a subset of *m* of the shares is selected. The method used for selecting which shares to use is arbitrary.

With the help of the function L(i)(U):

L(i)(U) = GF\_PROD(GF\_DIV(U[*j*], GF\_SUM(U[*j*], U[*i*])), *j*) for *j* = 0 … *m*, *j* ≠ *i*,

individual shares can be recombined into the secret by applying an interpolation function *I* (U, V) to an *m*-byte vector U and an *m*-byte input vector V:

 *I* (U, V) = GF\_SUM(X[*i*], *m*), where X[*i*] = GF\_PROD(L(i)(U), V[*i*]) for *i* = 0, *m* – 1.

The function L(i)(U) is defined as:

 L(i)(U) = GF\_PROD(GF\_DIV(U[*j*], GF\_SUM(U[*j*], U[*i*])), *j*) for *j* = 0 … *m*, *j* ≠ *i*

With input defined as:

* *m* = the threshold of shares required to be recombined to form the secret.
* *share*[*i*]= a vector of shares (*i* = 0 … *m*) with each share containing *L* + 1 bytes (note: the first byte contains *share\_id*[*i*]).

And with output defined as:

* *secret* = an *L*-byte vector containing the original secret.

The pseudo code algorithm for recombining the *n* shares to produce the original secret is:

**CheckErrors:**

 set length0 = length(share[0])

 **for** *i* = 0 to *m* - 1

 **do**

 **if** **length**(share[*i*]) **not equal** length0 then

 throw an error and exit

 **endif**

 **done**

**RecombineShares:**

 **for** *i* = 0 to *m* - 1

 **do**

 U[*i*] = share[*i*][0]

 **done**

 secret = **empty**

 **for** *i* = 0 to (length0 – 1)

 **do**

 **for** *j* = 0 to *m* - 1

 **do**

 V[j] = share[i][j + 1]

 **done**

 secret = secret **concatenate** *I*(U, V)

 **done**

## Constants

The following EXP and LOG tables are used for each defined polynomial.

### Polynomial 1

For {01} {1B} (283) (i.e., x8 + x4 + x3 + x + 1) the following tables apply.

Note: tables are defined such that LOG[EXP[*x*]] and EXP[LOG[*x*]] return *x* for all 0x00 < *x* ≤ 0xff.

#### LOG

0x00,0xff,0x19,0x01,0x32,0x02,0x1a,0xc6,

0x4b,0xc7,0x1b,0x68,0x33,0xee,0xdf,0x03,

0x64,0x04,0xe0,0x0e,0x34,0x8d,0x81,0xef,

0x4c,0x71,0x08,0xc8,0xf8,0x69,0x1c,0xc1,

0x7d,0xc2,0x1d,0xb5,0xf9,0xb9,0x27,0x6a,

0x4d,0xe4,0xa6,0x72,0x9a,0xc9,0x09,0x78,

0x65,0x2f,0x8a,0x05,0x21,0x0f,0xe1,0x24,

0x12,0xf0,0x82,0x45,0x35,0x93,0xda,0x8e,

0x96,0x8f,0xdb,0xbd,0x36,0xd0,0xce,0x94,

0x13,0x5c,0xd2,0xf1,0x40,0x46,0x83,0x38,

0x66,0xdd,0xfd,0x30,0xbf,0x06,0x8b,0x62,

0xb3,0x25,0xe2,0x98,0x22,0x88,0x91,0x10,

0x7e,0x6e,0x48,0xc3,0xa3,0xb6,0x1e,0x42,

0x3a,0x6b,0x28,0x54,0xfa,0x85,0x3d,0xba,

0x2b,0x79,0x0a,0x15,0x9b,0x9f,0x5e,0xca,

0x4e,0xd4,0xac,0xe5,0xf3,0x73,0xa7,0x57,

0xaf,0x58,0xa8,0x50,0xf4,0xea,0xd6,0x74,

0x4f,0xae,0xe9,0xd5,0xe7,0xe6,0xad,0xe8,

0x2c,0xd7,0x75,0x7a,0xeb,0x16,0x0b,0xf5,

0x59,0xcb,0x5f,0xb0,0x9c,0xa9,0x51,0xa0,

0x7f,0x0c,0xf6,0x6f,0x17,0xc4,0x49,0xec,

0xd8,0x43,0x1f,0x2d,0xa4,0x76,0x7b,0xb7,

0xcc,0xbb,0x3e,0x5a,0xfb,0x60,0xb1,0x86,

0x3b,0x52,0xa1,0x6c,0xaa,0x55,0x29,0x9d,

0x97,0xb2,0x87,0x90,0x61,0xbe,0xdc,0xfc,

0xbc,0x95,0xcf,0xcd,0x37,0x3f,0x5b,0xd1,

0x53,0x39,0x84,0x3c,0x41,0xa2,0x6d,0x47,

0x14,0x2a,0x9e,0x5d,0x56,0xf2,0xd3,0xab,

0x44,0x11,0x92,0xd9,0x23,0x20,0x2e,0x89,

0xb4,0x7c,0xb8,0x26,0x77,0x99,0xe3,0xa5,

0x67,0x4a,0xed,0xde,0xc5,0x31,0xfe,0x18,

0x0d,0x63,0x8c,0x80,0xc0,0xf7,0x70,0x07

#### EXP

0x01,0x03,0x05,0x0f,0x11,0x33,0x55,0xff,

0x1a,0x2e,0x72,0x96,0xa1,0xf8,0x13,0x35,

0x5f,0xe1,0x38,0x48,0xd8,0x73,0x95,0xa4,

0xf7,0x02,0x06,0x0a,0x1e,0x22,0x66,0xaa,

0xe5,0x34,0x5c,0xe4,0x37,0x59,0xeb,0x26,

0x6a,0xbe,0xd9,0x70,0x90,0xab,0xe6,0x31,

0x53,0xf5,0x04,0x0c,0x14,0x3c,0x44,0xcc,

0x4f,0xd1,0x68,0xb8,0xd3,0x6e,0xb2,0xcd,

0x4c,0xd4,0x67,0xa9,0xe0,0x3b,0x4d,0xd7,

0x62,0xa6,0xf1,0x08,0x18,0x28,0x78,0x88,

0x83,0x9e,0xb9,0xd0,0x6b,0xbd,0xdc,0x7f,

0x81,0x98,0xb3,0xce,0x49,0xdb,0x76,0x9a,

0xb5,0xc4,0x57,0xf9,0x10,0x30,0x50,0xf0,

0x0b,0x1d,0x27,0x69,0xbb,0xd6,0x61,0xa3,

0xfe,0x19,0x2b,0x7d,0x87,0x92,0xad,0xec,

0x2f,0x71,0x93,0xae,0xe9,0x20,0x60,0xa0,

0xfb,0x16,0x3a,0x4e,0xd2,0x6d,0xb7,0xc2,

0x5d,0xe7,0x32,0x56,0xfa,0x15,0x3f,0x41,

0xc3,0x5e,0xe2,0x3d,0x47,0xc9,0x40,0xc0,

0x5b,0xed,0x2c,0x74,0x9c,0xbf,0xda,0x75,

0x9f,0xba,0xd5,0x64,0xac,0xef,0x2a,0x7e,

0x82,0x9d,0xbc,0xdf,0x7a,0x8e,0x89,0x80,

0x9b,0xb6,0xc1,0x58,0xe8,0x23,0x65,0xaf,

0xea,0x25,0x6f,0xb1,0xc8,0x43,0xc5,0x54,

0xfc,0x1f,0x21,0x63,0xa5,0xf4,0x07,0x09,

0x1b,0x2d,0x77,0x99,0xb0,0xcb,0x46,0xca,

0x45,0xcf,0x4a,0xde,0x79,0x8b,0x86,0x91,

0xa8,0xe3,0x3e,0x42,0xc6,0x51,0xf3,0x0e,

0x12,0x36,0x5a,0xee,0x29,0x7b,0x8d,0x8c,

0x8f,0x8a,0x85,0x94,0xa7,0xf2,0x0d,0x17,

0x39,0x4b,0xdd,0x7c,0x84,0x97,0xa2,0xfd,

0x1c,0x24,0x6c,0xb4,0xc7,0x52,0xf6,0x01

### Polynomial 2

For {01} {1D} (285) (i.e., x8 + x4 + x3 + x2 + 1) the following tables apply.

Note: tables are defined such that LOG[EXP[*x*]] and EXP[LOG[*x*]] return *x* for all 0x00 < *x* ≤ 0xff.

#### LOG

0xff,0x00,0x01,0x19,0x02,0x32,0x1a,0xc6,

0x03,0xdf,0x33,0xee,0x1b,0x68,0xc7,0x4b,

0x04,0x64,0xe0,0x0e,0x34,0x8d,0xef,0x81,

0x1c,0xc1,0x69,0xf8,0xc8,0x08,0x4c,0x71,

0x05,0x8a,0x65,0x2f,0xe1,0x24,0x0f,0x21,

0x35,0x93,0x8e,0xda,0xf0,0x12,0x82,0x45,

0x1d,0xb5,0xc2,0x7d,0x6a,0x27,0xf9,0xb9,

0xc9,0x9a,0x09,0x78,0x4d,0xe4,0x72,0xa6,

0x06,0xbf,0x8b,0x62,0x66,0xdd,0x30,0xfd,

0xe2,0x98,0x25,0xb3,0x10,0x91,0x22,0x88,

0x36,0xd0,0x94,0xce,0x8f,0x96,0xdb,0xbd,

0xf1,0xd2,0x13,0x5c,0x83,0x38,0x46,0x40,

0x1e,0x42,0xb6,0xa3,0xc3,0x48,0x7e,0x6e,

0x6b,0x3a,0x28,0x54,0xfa,0x85,0xba,0x3d,

0xca,0x5e,0x9b,0x9f,0x0a,0x15,0x79,0x2b,

0x4e,0xd4,0xe5,0xac,0x73,0xf3,0xa7,0x57,

0x07,0x70,0xc0,0xf7,0x8c,0x80,0x63,0x0d,

0x67,0x4a,0xde,0xed,0x31,0xc5,0xfe,0x18,

0xe3,0xa5,0x99,0x77,0x26,0xb8,0xb4,0x7c,

0x11,0x44,0x92,0xd9,0x23,0x20,0x89,0x2e,

0x37,0x3f,0xd1,0x5b,0x95,0xbc,0xcf,0xcd,

0x90,0x87,0x97,0xb2,0xdc,0xfc,0xbe,0x61,

0xf2,0x56,0xd3,0xab,0x14,0x2a,0x5d,0x9e,

0x84,0x3c,0x39,0x53,0x47,0x6d,0x41,0xa2,

0x1f,0x2d,0x43,0xd8,0xb7,0x7b,0xa4,0x76,

0xc4,0x17,0x49,0xec,0x7f,0x0c,0x6f,0xf6,

0x6c,0xa1,0x3b,0x52,0x29,0x9d,0x55,0xaa,

0xfb,0x60,0x86,0xb1,0xbb,0xcc,0x3e,0x5a,

0xcb,0x59,0x5f,0xb0,0x9c,0xa9,0xa0,0x51,

0x0b,0xf5,0x16,0xeb,0x7a,0x75,0x2c,0xd7,

0x4f,0xae,0xd5,0xe9,0xe6,0xe7,0xad,0xe8,

0x74,0xd6,0xf4,0xea,0xa8,0x50,0x58,0xaf

#### EXP

0x01,0x02,0x04,0x08,0x10,0x20,0x40,0x80,

0x1d,0x3a,0x74,0xe8,0xcd,0x87,0x13,0x26,

0x4c,0x98,0x2d,0x5a,0xb4,0x75,0xea,0xc9,

0x8f,0x03,0x06,0x0c,0x18,0x30,0x60,0xc0,

0x9d,0x27,0x4e,0x9c,0x25,0x4a,0x94,0x35,

0x6a,0xd4,0xb5,0x77,0xee,0xc1,0x9f,0x23,

0x46,0x8c,0x05,0x0a,0x14,0x28,0x50,0xa0,

0x5d,0xba,0x69,0xd2,0xb9,0x6f,0xde,0xa1,

0x5f,0xbe,0x61,0xc2,0x99,0x2f,0x5e,0xbc,

0x65,0xca,0x89,0x0f,0x1e,0x3c,0x78,0xf0,

0xfd,0xe7,0xd3,0xbb,0x6b,0xd6,0xb1,0x7f,

0xfe,0xe1,0xdf,0xa3,0x5b,0xb6,0x71,0xe2,

0xd9,0xaf,0x43,0x86,0x11,0x22,0x44,0x88,

0x0d,0x1a,0x34,0x68,0xd0,0xbd,0x67,0xce,

0x81,0x1f,0x3e,0x7c,0xf8,0xed,0xc7,0x93,

0x3b,0x76,0xec,0xc5,0x97,0x33,0x66,0xcc,

0x85,0x17,0x2e,0x5c,0xb8,0x6d,0xda,0xa9,

0x4f,0x9e,0x21,0x42,0x84,0x15,0x2a,0x54,

0xa8,0x4d,0x9a,0x29,0x52,0xa4,0x55,0xaa,

0x49,0x92,0x39,0x72,0xe4,0xd5,0xb7,0x73,

0xe6,0xd1,0xbf,0x63,0xc6,0x91,0x3f,0x7e,

0xfc,0xe5,0xd7,0xb3,0x7b,0xf6,0xf1,0xff,

0xe3,0xdb,0xab,0x4b,0x96,0x31,0x62,0xc4,

0x95,0x37,0x6e,0xdc,0xa5,0x57,0xae,0x41,

0x82,0x19,0x32,0x64,0xc8,0x8d,0x07,0x0e,

0x1c,0x38,0x70,0xe0,0xdd,0xa7,0x53,0xa6,

0x51,0xa2,0x59,0xb2,0x79,0xf2,0xf9,0xef,

0xc3,0x9b,0x2b,0x56,0xac,0x45,0x8a,0x09,

0x12,0x24,0x48,0x90,0x3d,0x7a,0xf4,0xf5,

0xf7,0xf3,0xfb,0xeb,0xcb,0x8b,0x0b,0x16,

0x2c,0x58,0xb0,0x7d,0xfa,0xe9,0xcf,0x83,

0x1b,0x36,0x6c,0xd8,0xad,0x47,0x8e,0x01

## Test Vectors

The following test vectors are defined for each defined polynomial.

### Polynomial 1

For {01} {1B} (283) (i.e., x8 + x4 + x3 + x + 1) the following tests apply.

Note: in the test vectors, the share splits are defined without the leading share ID. The share ID associated with a share are indicated separately.

#### Test Vector TV011B\_1

secret=7465737400

m=2

n=2

random=A87B3491B58BE31DBA42

split1=DC1E47E5B5 (share ID = 0x01)

split2=3F931B4D71 (share ID = 0x02)

#### Test Vector TV011B\_2

secret=53414D5443

m=2

n=4

random=395D396C87CE75528C6691C7C0BF233AE7C96F8B

split1=6A1C7438C4 (share ID = 0x01)

split2=21FB3F8C56 (share ID = 0x02)

split3=18A606E0D1 (share ID = 0x03)

split4=B72EA9FF69 (share ID = 0x04)

#### Test Vector TV011B\_3

secret=53414D5443

m=3

n=4

random=273A1A28AB9979BC06377C48595FF88195C2FB50

split1=4E737F9172 (share ID = 0x01)

split2=F5D5526093 (share ID = 0x02)

split3=E8E760A5A2 (share ID = 0x03)

split4=429F849E06 (share ID = 0x04)

#### Test Vector TV011B\_4

secret=53414D5443

m=4

n=4

random=1A224C1EE9760A73A09D05774434672361D9ADF5

split1=27C094BB54 (share ID = 0x01)

split2=B969F9F40E (share ID = 0x02)

split3=7EC7CD3250 (share ID = 0x03)

split4=ABAF81828D (share ID = 0x04)

#### Test Vector TV011B\_5

secret=546573742044617461

m=2

n=9

random=7FB4E8581EB75DC9453FE7837DDC3036F50347B820B85C88059274BA279E5EF11DF69B841F9A2636C7AA3613EED2B8C94F8C12A3C0D99603BFA9CB7B7CFDF81927B1E01B162ACF119BDE67FE7A6C9BDC7F

split1=2BD19B2C3EF33CBD24 (share ID = 0x01)

split2=AA16B8C41C31DBFDEB (share ID = 0x02)

split3=D5A2509C02868634AE (share ID = 0x03)

split4=B383FE0F58AE0E7D6E (share ID = 0x04)

split5=CC371657461953B42B (share ID = 0x05)

split6=4DF035BF64DBB4F4E4 (share ID = 0x06)

split7=3244DDE77A6CE93DA1 (share ID = 0x07)

split8=81B27282D08BBF667F (share ID = 0x08)

split9=FE069ADACE3CE2AF3A (share ID = 0x09)

#### Test Vector TV011B\_6

secret=0102030405060708090A0B0C0D0E0F

m=3

n=5

random=EC96740540B3E1FC9A914F6E5F7CCA51DB723202C9B881004F66A28071974F36070E5A77DA6F79355B5A3D9CB932DE31B247766B3F30E002FBFE9B415708F9811DE4C1011F13291AFF41D4

split1=7B73F0190E272493A03A7A8D242CE9 (share ID = 0x01)

split2=ACFE7900583B52D877665415106787 (share ID = 0x02)

split3=D68F8A1D531A7143DE562594394561 (share ID = 0x03)

split4=3F99DDF4889BE16A29E2773E106863 (share ID = 0x04)

split5=45E82EE983BAC2F180D206BF394A85 (share ID = 0x05)

### Polynomial 2

For {01} {1D} (285) (i.e., x8 + x4 + x3 + x2 + 1) the following tests apply.

Note: in the test vectors, the share splits are defined without the leading share ID. The share ID associated with a share are indicated separately.

#### Test Vector TV011D\_1

secret=7465737400

m=2

n=2

random=F3C23381F59F58B39A8B

split1=87A740F5F5 (share ID = 0x01)

split2=8FFC156BF7 (share ID = 0x02)

#### Test Vector TV011D\_2

secret=53414D5443

m=2

n=4

random=207608930C013A33DD325324F583054DD8A0C03D

split1=733745C74F (share ID = 0x01)

split2=13AD5D6F5B (share ID = 0x02)

split3=33DB55FC57 (share ID = 0x03)

split4=D3846D2273 (share ID = 0x04)

#### Test Vector TV011D\_3

secret=53414D5443

m=3

n=4

random=8C1592625C4AAF5341452253A3FBD2E791EFF389

split1=CAB15BA847 (share ID = 0x01)

split2=02EDC046C8 (share ID = 0x02)

split3=9B1DD6BACC (share ID = 0x03)

split4=145DF48B7E (share ID = 0x04)

#### Test Vector TV011D\_4

secret=53414D5443

m=4

n=4

random=72B0883C96B9CBB9CBB28266F379FA4B7501A26A

split1=1952F40233 (share ID = 0x01)

split2=79FA0E08C2 (share ID = 0x02)

split3=2458371794 (share ID = 0x03)

split4=F445A9D607 (share ID = 0x04)

#### Test Vector TV011D\_5

secret=546573742044617461

m=2

n=9

random=AFFD2B0BFA3433639CE8ECCEEA27F21AA5C1F562D2E090F2B7E7DFD62F0C48A82ADC79876594E020D2499F6049A551D68ED675EF05557EC8FB0C87DFCE4673299A877269B8879F322DE8F25A9A283C9DCE

split1=FB98587FDA705217FD (share ID = 0x01)

split2=17822562C92C07B244 (share ID = 0x02)

split3=B87F0E69331834D1D8 (share ID = 0x03)

split4=D2B6DF58EF94ADE52B (share ID = 0x04)

split5=7D4BF45315A09E86B7 (share ID = 0x05)

split6=9151894E06FCCB230E (share ID = 0x06)

split7=3EACA245FCC8F84092 (share ID = 0x07)

split8=45DE362CA3F9E44BF5 (share ID = 0x08)

split9=EA231D2759CDD72869 (share ID = 0x09)

#### Test Vector TV011D\_6

secret=0102030405060708090A0B0C0D0E0F

m=3

n=5

random=024A89AC968C986577FEB024116B94F654DDDE209C3CC3E448884D31F8C8D8A8074C8B900B062AF42C857A6D4EB2BAC11F33BB92C8A74A1FE20C2F550E902F405B43C0E8E22E4CA91B7D55

split1=492719F98C927D6A80F4AB2BCD723F (share ID = 0x01)

split2=308738A034EB94C2F22BDE208750E5 (share ID = 0x02)

split3=78A2225DBD7FEEA07BD57E07472CD5 (share ID = 0x03)

split4=DD0E49409F86BDB9156FA6C15810D4 (share ID = 0x04)

split5=952B53BD1612C7DB9C9106E6986CE4 (share ID = 0x05)

# Conformance

## SAM Threshold Sharing Scheme 1 (TSS1) Implementation Conformance

An implementation is a conforming SAM TSS1 implementation if the implementation passes all applicable Test Cases in this document.

A SAM TSS1 implementation SHALL be a conforming SAM TSS1 implementation.

1. References

This appendix contains the normative and informative references that are used in this document. Normative references are specific (identified by date of publication and/or edition number or Version number) and Informative references are either specific or non-specific.

While any hyperlinks included in this appendix were valid at the time of publication, OASIS cannot guarantee their long-term validity.

* 1. Normative References

The following documents are referenced in such a way that some or all of their content constitutes requirements of this document.

[FIPS197] Advanced Encryption Standard, FIPS PUB 197, Nov 2001, <https://csrc.nist.gov/csrc/media/publications/fips/197/final/documents/fips-197.pdf>

[RFC2119] Bradner, S., "Key words for use in RFCs to Indicate Requirement Levels", BCP 14, RFC 2119, DOI 10.17487/RFC2119, March 1997, <<https://www.rfc-editor.org/info/rfc2119>>.

**[****RFC8174]** Leiba, B., "Ambiguity of Uppercase vs Lowercase in RFC 2119 Key Words", BCP 14, RFC 8174, DOI 10.17487/RFC8174, May 2017, <<https://www.rfc-editor.org/info/rfc8174>>.

**[****SHAMIR79]** Shamir, Adi, "How to share a secret", Communications of the ACM, 22(11) : 612-613, <<https://dl.acm.org/doi/10.1145/359168.359176>>.

* 1. Informative References

The following referenced documents are not required for the application of this document but may assist the reader with regard to a particular subject area.

**[****Cryptol]** Galois, Inc., <<https://www.cryptol.net>>.

1. Acknowledgments

* 1. Participants

The following individuals were members of this Technical Committee during the creation of this document and their contributions are gratefully acknowledged:

[Participant Name, Affiliation | Individual Member]

| **Person** | **Organization** | **Role** |
| --- | --- | --- |
| Patrick Maroney | AT&T | Member |
| Tony Cox | Cryptsoft Pty Ltd. | Chair |
| Tim Hudson | Cryptsoft Pty Ltd. | Voting Member |
| Scott Marshall | Cryptsoft Pty Ltd. | Voting Member |
| Bruce Rich | Cryptsoft Pty Ltd. | Voting Member |
| Greg Scott | Cryptsoft Pty Ltd. | Voting Member |
| Jason Thatcher | Cryptsoft Pty Ltd. | Voting Member |
| Judith Furlong | Dell | Voting Member |
| Deepak Gaikwad | Dell | Voting Member |
| Ravi Sharda | Dell | Member |
| Gerald Stueve | Fornetix | Voting Member |
| Christopher Hillier | Hewlett Packard Enterprise (HPE) | Member |
| Andrea Caccia | Individual | Voting Member |
| Sander Fieten | Individual | Member |
| Stefan Hagen | Individual | Secretary |
| Tim Chevalier | NetApp | Chair |
| Pim van der Eijk | Sonnenglanz Consulting | Member |

1. Revision History

Revisions made since the initial stage of this numbered Version of this document may be tracked here.

|  |  |  |  |
| --- | --- | --- | --- |
| **Revision** | **Date** | **Editor** | **Changes Made** |
| CSD01 | May 2, 2021 | Tim Chevalier and Tim Hudson | Initial draft |
| CS01 | August 4, 2021 | TC Admin |  Published as Committee Specification 01 |
| CS01 (update) | August 11, 2021 | Tony Cox | Added “Informative” notation to Appendix E |
| CS02 | August 19, 2021 | Tony Cox | Published CS |
| CS02 Update | September 21, 2021 | Tim Chevalier and Tim Hudson | CS02 with non-material changes following further editorial review |

1. Secret Sharing described in terms of matrix operations

The secret share creation process and the secret share recombination process may be described as matrix operations over GF(256).

* 1. Create Shares

Given a secret, *s*, of length *L* bytes, share creation can be expressed as matrix multiplication over GF(256):

*P* *S* = **

where:

*P* is an *n x m* matrix,

S is an *m x L* matrix,

 is an *n x L* matrix containing the shares,

*n* = number of shares to be created, and

*m* = the threshold of shares required to be recombined to form the secret.

* + 1. Creating the P matrix

The *Pij* elements contained in the *n x m* matrix *P* are given by the following formula:

*Pij* = GF\_POW(*i*, *j* – 1) for *i* = 1..*n* and *j* = 1..*m*

Where GF\_POW(*i*, *j* - 1) is defined as the GF(256) element *i* raised to the power (*j* - 1). Thus, *P* contains the follow elements:

*Pi*,1 = GF\_POW(*i*, 0x00) = 0x01

*Pi,2 =* GF\_POW(*i*, 0x01) = GF\_MUL(*i*, *Pi,1*) = EXP[LOG[*i*]] = *i*

*Pi,3* = GF\_POW(*i*, 0x02) = GF\_MUL(*i*, *Pi,2*) = EXP[(LOG[*i*] + LOG[*Pi,2*]) % 255]

*Pi,4 =* GF\_POW(*i*, 0x03) = GF\_MUL(*i*, *Pi,3*) = EXP[(LOG[*i*] + LOG[*Pi,3*]) % 255]

*…*

*Pi,m =* GF\_POW(*i*, *m* - 1) = GF\_MUL(*i*, *Pi,m-1*) = EXP[(LOG[*i*] + LOG[*Pi,m-1*]) % 255]

* + 1. Creating the S matrix

The *S* matrix (with dimensions *m* x *L*) is created from the secret, *s* (length *L*), and a random set of bytes *R* of length (*m* – 1) *L*. The elements of the matrix S are:

*S1,j* = byte *s*[*j*] from secret *s* for *j* = 1, …, *L*

*Si,j* = byte *R*[*k*], where *k* = (*i* – 1) + (*m* – 1)(*j* – 1), for *i* = 2, …, m and j = 1, …, *L*

For example, with:

*s* = [ 53 41 4D 54 43]

*m* = 4

*n* = 4, and

R = [1A 22 4C 1E E9 76 0A 73 A0 9D 05 77 44 34 67]

The (4 x 5) matrix *S* would be:

 53 41 4D 54 43

 1A 1E 0A 9D 44

 22 E9 73 05 34

 4C 76 A0 77 67

* + 1. Calculating the  matrix

The  matrix is calculated as the matrix product *PS*, with individual elements of  calculated as the dot product over GF(128):

*i,j* = *Pi,1S1,j* + *Pi,2S2,j* + … + *Pn,mSm,l*

 = GF\_SUM[*Pi,1S1,j*, *Pi,2S2,j*, …, *Pn,mSm,l*]

 = GF\_SUM[GF\_MUL(*Pi,1*,*S1,j*), GF\_MUL(*Pi,2*, *1S2,j*), … , GF\_MUL(*Pn,m*, *Sm,l*)]

* 1. Recombine Shares

Given *m* share indices, where each share index, U*i*, is unique and 1 ≤ U*i* ≤ 255, share recombination can be expressed as matrix multiplication over GF(256):

 L  = s

where:

L is a 1 x *m* matrix,

 is an *m* x *L* matrix containing shares,

*m* = threshold number of shares to be recombined, and

s = the secret (of length *L*).

The L i*j* elements contained in the 1 x *m* matrix L are given by the following formula:

L *ij* = GF\_PROD(GF\_DIV(Uj, GF\_SUM(Uj, Ui)) for *j* = 0...*m* – 1, *j* ≠ *i*

1. Cryptol code for testing secret share creation and recombination

This appendix is non-normative (informative).

Cryptol [[Cryptol](#Cryptol)] is a programming language used for cryptography that is developed and maintained by Galois, Inc.. This appendix contains Cryptol code for creating and recombining splits for both the polynomial and the polynomial. In addition, there are separate table-based and “native” GF(256)-based implementations.

* 1. Polynomial x8 + x4 + x3 + x + 1

Cryptol code for creating splits, checking the splits, re-combining splits, and checking the recombinations appears in the following subsections.

* + 1. Table-based implementation

module secret\_share\_011B\_table where

type GF28 = [8]

/\* given an alpha^j, where alpha = mimimum primitive element (x + 1 = 3), return j \*/

LOG : [256][8]

LOG = [

 0x00, 0xff, 0x19, 0x01, 0x32, 0x02, 0x1a, 0xc6,

 0x4b, 0xc7, 0x1b, 0x68, 0x33, 0xee, 0xdf, 0x03,

 0x64, 0x04, 0xe0, 0x0e, 0x34, 0x8d, 0x81, 0xef,

 0x4c, 0x71, 0x08, 0xc8, 0xf8, 0x69, 0x1c, 0xc1,

 0x7d, 0xc2, 0x1d, 0xb5, 0xf9, 0xb9, 0x27, 0x6a,

 0x4d, 0xe4, 0xa6, 0x72, 0x9a, 0xc9, 0x09, 0x78,

 0x65, 0x2f, 0x8a, 0x05, 0x21, 0x0f, 0xe1, 0x24,

 0x12, 0xf0, 0x82, 0x45, 0x35, 0x93, 0xda, 0x8e,

 0x96, 0x8f, 0xdb, 0xbd, 0x36, 0xd0, 0xce, 0x94,

 0x13, 0x5c, 0xd2, 0xf1, 0x40, 0x46, 0x83, 0x38,

 0x66, 0xdd, 0xfd, 0x30, 0xbf, 0x06, 0x8b, 0x62,

 0xb3, 0x25, 0xe2, 0x98, 0x22, 0x88, 0x91, 0x10,

 0x7e, 0x6e, 0x48, 0xc3, 0xa3, 0xb6, 0x1e, 0x42,

 0x3a, 0x6b, 0x28, 0x54, 0xfa, 0x85, 0x3d, 0xba,

 0x2b, 0x79, 0x0a, 0x15, 0x9b, 0x9f, 0x5e, 0xca,

 0x4e, 0xd4, 0xac, 0xe5, 0xf3, 0x73, 0xa7, 0x57,

 0xaf, 0x58, 0xa8, 0x50, 0xf4, 0xea, 0xd6, 0x74,

 0x4f, 0xae, 0xe9, 0xd5, 0xe7, 0xe6, 0xad, 0xe8,

 0x2c, 0xd7, 0x75, 0x7a, 0xeb, 0x16, 0x0b, 0xf5,

 0x59, 0xcb, 0x5f, 0xb0, 0x9c, 0xa9, 0x51, 0xa0,

 0x7f, 0x0c, 0xf6, 0x6f, 0x17, 0xc4, 0x49, 0xec,

 0xd8, 0x43, 0x1f, 0x2d, 0xa4, 0x76, 0x7b, 0xb7,

 0xcc, 0xbb, 0x3e, 0x5a, 0xfb, 0x60, 0xb1, 0x86,

 0x3b, 0x52, 0xa1, 0x6c, 0xaa, 0x55, 0x29, 0x9d,

 0x97, 0xb2, 0x87, 0x90, 0x61, 0xbe, 0xdc, 0xfc,

 0xbc, 0x95, 0xcf, 0xcd, 0x37, 0x3f, 0x5b, 0xd1,

 0x53, 0x39, 0x84, 0x3c, 0x41, 0xa2, 0x6d, 0x47,

 0x14, 0x2a, 0x9e, 0x5d, 0x56, 0xf2, 0xd3, 0xab,

 0x44, 0x11, 0x92, 0xd9, 0x23, 0x20, 0x2e, 0x89,

 0xb4, 0x7c, 0xb8, 0x26, 0x77, 0x99, 0xe3, 0xa5,

 0x67, 0x4a, 0xed, 0xde, 0xc5, 0x31, 0xfe, 0x18,

 0x0d, 0x63, 0x8c, 0x80, 0xc0, 0xf7, 0x70, 0x07

]

/\* given a j, return alpha^j, where alpha = mimimum primitive element (x + 1 = 3) \*/

EXP : [256][8]

EXP = [

 0x01, 0x03, 0x05, 0x0f, 0x11, 0x33, 0x55, 0xff,

 0x1a, 0x2e, 0x72, 0x96, 0xa1, 0xf8, 0x13, 0x35,

 0x5f, 0xe1, 0x38, 0x48, 0xd8, 0x73, 0x95, 0xa4,

 0xf7, 0x02, 0x06, 0x0a, 0x1e, 0x22, 0x66, 0xaa,

 0xe5, 0x34, 0x5c, 0xe4, 0x37, 0x59, 0xeb, 0x26,

 0x6a, 0xbe, 0xd9, 0x70, 0x90, 0xab, 0xe6, 0x31,

 0x53, 0xf5, 0x04, 0x0c, 0x14, 0x3c, 0x44, 0xcc,

 0x4f, 0xd1, 0x68, 0xb8, 0xd3, 0x6e, 0xb2, 0xcd,

 0x4c, 0xd4, 0x67, 0xa9, 0xe0, 0x3b, 0x4d, 0xd7,

 0x62, 0xa6, 0xf1, 0x08, 0x18, 0x28, 0x78, 0x88,

 0x83, 0x9e, 0xb9, 0xd0, 0x6b, 0xbd, 0xdc, 0x7f,

 0x81, 0x98, 0xb3, 0xce, 0x49, 0xdb, 0x76, 0x9a,

 0xb5, 0xc4, 0x57, 0xf9, 0x10, 0x30, 0x50, 0xf0,

 0x0b, 0x1d, 0x27, 0x69, 0xbb, 0xd6, 0x61, 0xa3,

 0xfe, 0x19, 0x2b, 0x7d, 0x87, 0x92, 0xad, 0xec,

 0x2f, 0x71, 0x93, 0xae, 0xe9, 0x20, 0x60, 0xa0,

 0xfb, 0x16, 0x3a, 0x4e, 0xd2, 0x6d, 0xb7, 0xc2,

 0x5d, 0xe7, 0x32, 0x56, 0xfa, 0x15, 0x3f, 0x41,

 0xc3, 0x5e, 0xe2, 0x3d, 0x47, 0xc9, 0x40, 0xc0,

 0x5b, 0xed, 0x2c, 0x74, 0x9c, 0xbf, 0xda, 0x75,

 0x9f, 0xba, 0xd5, 0x64, 0xac, 0xef, 0x2a, 0x7e,

 0x82, 0x9d, 0xbc, 0xdf, 0x7a, 0x8e, 0x89, 0x80,

 0x9b, 0xb6, 0xc1, 0x58, 0xe8, 0x23, 0x65, 0xaf,

 0xea, 0x25, 0x6f, 0xb1, 0xc8, 0x43, 0xc5, 0x54,

 0xfc, 0x1f, 0x21, 0x63, 0xa5, 0xf4, 0x07, 0x09,

 0x1b, 0x2d, 0x77, 0x99, 0xb0, 0xcb, 0x46, 0xca,

 0x45, 0xcf, 0x4a, 0xde, 0x79, 0x8b, 0x86, 0x91,

 0xa8, 0xe3, 0x3e, 0x42, 0xc6, 0x51, 0xf3, 0x0e,

 0x12, 0x36, 0x5a, 0xee, 0x29, 0x7b, 0x8d, 0x8c,

 0x8f, 0x8a, 0x85, 0x94, 0xa7, 0xf2, 0x0d, 0x17,

 0x39, 0x4b, 0xdd, 0x7c, 0x84, 0x97, 0xa2, 0xfd,

 0x1c, 0x24, 0x6c, 0xb4, 0xc7, 0x52, 0xf6, 0x01

]

GF\_ADD : (GF28, GF28) -> GF28

GF\_ADD (x, y) = x ^ y

GF\_SUB : (GF28, GF28) -> GF28

GF\_SUB (x, y) = x ^ y

GF\_MUL : (GF28, GF28) -> GF28

GF\_MUL (x, y) = z

 where z = if x == 0 then 0

 else if y == 0 then 0

 else EXP @ ((toInteger (LOG @ x) + toInteger (LOG @ y)) % 255)

GF\_POW : (GF28, [8]) -> GF28

GF\_POW (n, k) = pows ! 0

 where pows = [1] # [ if bit then GF\_MUL (n, sq x)

 else sq x

 | x <- pows

 | bit <- k

 ]

 sq x = GF\_MUL (x, x)

GF\_DIV : (GF28, GF28) -> GF28

GF\_DIV (x, y) = div x

 where div i = if x == 0 then 0

 else EXP @ (((toInteger(LOG @ x)) - toInteger (LOG @ y)) % 255)

GF\_SUM : {n} (fin n) => [n]GF28 -> GF28

GF\_SUM ps = accum ! 0

 where accum = [zero] # [ GF\_ADD(p, a) | p <- ps | a <- accum ]

GF\_PROD : {n} (fin n) => [n]GF28 -> GF28

GF\_PROD ps = accum ! 0

 where accum = [1] # [ GF\_MUL(p, a) | p <- ps | a <- accum ]

GF\_DOT\_PROD : {n} (fin n) => ([n]GF28, [n]GF28) -> GF28

GF\_DOT\_PROD (xs, ys) = GF\_SUM [ GF\_MUL (x, y) | x <- xs

 | y <- ys

 ]

GF\_VEC\_MUL : {n, m} (fin n) => ([n]GF28, [m][n]GF28) -> [m]GF28

GF\_VEC\_MUL (v, ms) = [ GF\_DOT\_PROD(v, m) | m <- ms ]

GF\_MAT\_MUL : {n, m, k} (fin m) => ([n][m]GF28, [m][k]GF28) -> [n][k]GF28

GF\_MAT\_MUL (xss, yss) = [ GF\_VEC\_MUL(xs, yss') | xs <- xss ]

 where yss' = transpose yss

// Test test vectors for Polynomial 1 (x^^8 + x^^4 + x^^3 + x + 1)

/\*

 \* Test vector TV011B\_1

 \* secret = 74 65 73 74 00

 \* random = A8 7B 34 91 B5

 \*

 \* l = 5

 \* m = 2

 \* n = 2

 \*

 \* split1 = DC 1E 47 E5 B5

 \* split2 = 3F 93 1B 4D 71

 \*/

TV011B\_TV1\_P : [2][2]GF28

TV011B\_TV1\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01) ]

]

TV011B\_TV1\_SR : [2][5]GF28

TV011B\_TV1\_SR = [

 [ 0x74, 0x65, 0x73, 0x74, 0x00 ],

 [ 0xA8, 0x7B, 0x34, 0x91, 0xB5 ]

]

TV011B\_TV1\_SPLITS : [2][5]GF28

TV011B\_TV1\_SPLITS = [

 [ 0xDC, 0x1E, 0x47, 0xE5, 0xB5 ],

 [ 0x3F, 0x93, 0x1B, 0x4D, 0x71 ]

]

/\* test split \*/

property TV011B\_TV1\_split\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV1\_P, TV011B\_TV1\_SR) == TV011B\_TV1\_SPLITS ] == ~zero

TV011B\_TV1\_1\_2\_R : [1][2]GF28

TV011B\_TV1\_1\_2\_R = [

 [ GF\_DIV(0x02, GF\_ADD(0x02, 0x01)), GF\_DIV(0x01, GF\_ADD(0x01, 0x02)) ]

]

TV011B\_TV1\_1\_2\_SPLITS : [2][5]GF28

TV011B\_TV1\_1\_2\_SPLITS = [

 [ 0xDC, 0x1E, 0x47, 0xE5, 0xB5 ],

 [ 0x3F, 0x93, 0x1B, 0x4D, 0x71 ]

]

TV011B\_TV1\_SECRET : [1][5]GF28

TV011B\_TV1\_SECRET = [

 [ 0x74, 0x65, 0x73, 0x74, 0x00 ]

]

/\* test recombine \*/

property TV011B\_TV1\_recombine\_1\_2\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV1\_1\_2\_R, TV011B\_TV1\_1\_2\_SPLITS) == TV011B\_TV1\_SECRET ] == ~zero

/\*

 \* Test vector TV011B\_2

 \* secret = 53 41 4D 54 43

 \* random = 39 5D 39 6C 87

 \*

 \* l = 5

 \* m = 2

 \* n = 4

 \*

 \* split1 = 6A 1C 74 38 C4

 \* split2 = 21 FB 3F 8C 56

 \* split3 = 18 A6 06 E0 D1

 \* split4 = B7 2E A9 FF 69

 \*/

TV011B\_TV2\_P : [4][2]GF28

TV011B\_TV2\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01) ]

]

TV011B\_TV2\_SR : [2][5]GF28

TV011B\_TV2\_SR = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ],

 [ 0x39, 0x5D, 0x39, 0x6C, 0x87 ]

]

TV011B\_TV2\_SPLITS : [4][5]GF28

TV011B\_TV2\_SPLITS = [

 [ 0x6A, 0x1C, 0x74, 0x38, 0xC4 ],

 [ 0x21, 0xFB, 0x3F, 0x8C, 0x56 ],

 [ 0x18, 0xA6, 0x06, 0xE0, 0xD1 ],

 [ 0xB7, 0x2E, 0xA9, 0xFF, 0x69 ]

]

/\* test split \*/

property TV011B\_TV2\_split\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV2\_P, TV011B\_TV2\_SR) == TV011B\_TV2\_SPLITS ] == ~zero

TV011B\_TV2\_1\_2\_R : [1][2]GF28

TV011B\_TV2\_1\_2\_R = [

 [ GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x01, GF\_ADD(0x01, 0x02)) ]

]

TV011B\_TV2\_1\_4\_R : [1][2]GF28

TV011B\_TV2\_1\_4\_R = [

 [ GF\_DIV(0x04, GF\_ADD(0x01, 0x04)), GF\_DIV(0x01, GF\_ADD(0x01, 0x04)) ]

]

TV011B\_TV2\_3\_4\_R : [1][2]GF28

TV011B\_TV2\_3\_4\_R = [

 [ GF\_DIV(0x04, GF\_ADD(0x03, 0x04)), GF\_DIV(0x03, GF\_ADD(0x03, 0x04)) ]

]

TV011B\_TV2\_1\_2\_SPLITS : [2][5]GF28

TV011B\_TV2\_1\_2\_SPLITS = [

 [ 0x6A, 0x1C, 0x74, 0x38, 0xC4 ],

 [ 0x21, 0xFB, 0x3F, 0x8C, 0x56 ]

]

TV011B\_TV2\_1\_4\_SPLITS : [2][5]GF28

TV011B\_TV2\_1\_4\_SPLITS = [

 [ 0x6A, 0x1C, 0x74, 0x38, 0xC4 ],

 [ 0xB7, 0x2E, 0xA9, 0xFF, 0x69 ]

]

TV011B\_TV2\_3\_4\_SPLITS : [2][5]GF28

TV011B\_TV2\_3\_4\_SPLITS = [

 [ 0x18, 0xA6, 0x06, 0xE0, 0xD1 ],

 [ 0xB7, 0x2E, 0xA9, 0xFF, 0x69 ]

]

TV011B\_TV2\_SECRET : [1][5]GF28

TV011B\_TV2\_SECRET = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ]

]

/\* test recombines \*/

property TV011B\_TV2\_recombine\_1\_2\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV2\_1\_2\_R, TV011B\_TV2\_1\_2\_SPLITS) == TV011B\_TV2\_SECRET ] == ~zero

property TV011B\_TV2\_recombine\_1\_4\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV2\_1\_4\_R, TV011B\_TV2\_1\_4\_SPLITS) == TV011B\_TV2\_SECRET ] == ~zero

property TV011B\_TV2\_recombine\_3\_4\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV2\_3\_4\_R, TV011B\_TV2\_3\_4\_SPLITS) == TV011B\_TV2\_SECRET ] == ~zero

/\*

 \* Test vector TV011B\_3

 \* secret = 53 41 4D 54 43

 \* random = 27 3A 1A 28 AB 99 79 BC 06 37

 \*

 \* l = 5

 \* m = 3

 \* n = 4

 \*

 \* split1 = 4E 73 7F 91 72

 \* split2 = F5 D5 52 60 93

 \* split3 = E8 E7 60 A5 A2

 \* split4 = 42 9F 84 9E 06

 \*/

TV011B\_TV3\_P : [4][3]GF28

TV011B\_TV3\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01), GF\_POW(0x01, 0x02) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01), GF\_POW(0x02, 0x02) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01), GF\_POW(0x03, 0x02) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01), GF\_POW(0x04, 0x02) ]

]

TV011B\_TV3\_SR : [3][5]GF28

TV011B\_TV3\_SR = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ],

 [ 0x27, 0x1A, 0xAB, 0x79, 0x06 ],

 [ 0x3A, 0x28, 0x99, 0xBC, 0x37 ]

]

TV011B\_TV3\_SPLITS : [4][5]GF28

TV011B\_TV3\_SPLITS = [

 [ 0x4E, 0x73, 0x7F, 0x91, 0x72 ],

 [ 0xF5, 0xD5, 0x52, 0x60, 0x93 ],

 [ 0xE8, 0xE7, 0x60, 0xA5, 0xA2 ],

 [ 0x42, 0x9F, 0x84, 0x9E, 0x06 ]

]

/\* test split \*/

property TV011B\_TV3\_split\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV3\_P, TV011B\_TV3\_SR) == TV011B\_TV3\_SPLITS ] == ~zero

TV011B\_TV3\_1\_2\_3\_R : [1][3]GF28

TV011B\_TV3\_1\_2\_3\_R = [

 [ GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x01, 0x03))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x02, 0x03))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x03)), GF\_DIV(0x02, GF\_ADD(0x02, 0x03))]

 ]

]

TV011B\_TV3\_1\_2\_4\_R : [1][3]GF28

TV011B\_TV3\_1\_2\_4\_R = [

 [ GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x04, GF\_ADD(0x01, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x02)), GF\_DIV(0x04, GF\_ADD(0x02, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x04)), GF\_DIV(0x02, GF\_ADD(0x02, 0x04))]

 ]

]

TV011B\_TV3\_1\_3\_4\_R : [1][3]GF28

TV011B\_TV3\_1\_3\_4\_R = [

 [ GF\_PROD[GF\_DIV(0x03, GF\_ADD(0x01, 0x03)), GF\_DIV(0x04, GF\_ADD(0x01, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x03)), GF\_DIV(0x04, GF\_ADD(0x03, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x04)), GF\_DIV(0x03, GF\_ADD(0x03, 0x04))]

 ]

]

TV011B\_TV3\_1\_2\_3\_SPLITS : [3][5]GF28

TV011B\_TV3\_1\_2\_3\_SPLITS = [

 [ 0x4E, 0x73, 0x7F, 0x91, 0x72 ],

 [ 0xF5, 0xD5, 0x52, 0x60, 0x93 ],

 [ 0xE8, 0xE7, 0x60, 0xA5, 0xA2 ]

]

TV011B\_TV3\_1\_2\_4\_SPLITS : [3][5]GF28

TV011B\_TV3\_1\_2\_4\_SPLITS = [

 [ 0x4E, 0x73, 0x7F, 0x91, 0x72 ],

 [ 0xF5, 0xD5, 0x52, 0x60, 0x93 ],

 [ 0x42, 0x9F, 0x84, 0x9E, 0x06 ]

]

TV011B\_TV3\_1\_3\_4\_SPLITS : [3][5]GF28

TV011B\_TV3\_1\_3\_4\_SPLITS = [

 [ 0x4E, 0x73, 0x7F, 0x91, 0x72 ],

 [ 0xE8, 0xE7, 0x60, 0xA5, 0xA2 ],

 [ 0x42, 0x9F, 0x84, 0x9E, 0x06 ]

]

TV011B\_TV3\_SECRET : [1][5]GF28

TV011B\_TV3\_SECRET = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ]

]

/\* test recombines \*/

property TV011B\_TV3\_recombine\_1\_2\_3\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV3\_1\_2\_3\_R, TV011B\_TV3\_1\_2\_3\_SPLITS) == TV011B\_TV3\_SECRET ] == ~zero

property TV011B\_TV3\_recombine\_1\_2\_4\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV3\_1\_2\_4\_R, TV011B\_TV3\_1\_2\_4\_SPLITS) == TV011B\_TV3\_SECRET ] == ~zero

property TV011B\_TV3\_recombine\_1\_3\_4\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV3\_1\_3\_4\_R, TV011B\_TV3\_1\_3\_4\_SPLITS) == TV011B\_TV3\_SECRET ] == ~zero

/\*

 \* Test vector TV011B\_4

 \* secret = 53 41 4D 54 43

 \* random = 1A 22 4C 1E E9 76 0A 73 A0 9D 05 77 44 34 67

 \*

 \* l = 5

 \* m = 4

 \* n = 4

 \*

 \* split1 = 27 C0 94 BB 54

 \* split2 = B9 69 F9 F4 0E

 \* split3 = 7E C7 CD 32 50

 \* split4 = AB AF 81 82 8D

 \*/

TV011B\_TV4\_P : [4][4]GF28

TV011B\_TV4\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01), GF\_POW(0x01, 0x02), GF\_POW(0x01, 0x03) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01), GF\_POW(0x02, 0x02), GF\_POW(0x02, 0x03) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01), GF\_POW(0x03, 0x02), GF\_POW(0x03, 0x03) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01), GF\_POW(0x04, 0x02), GF\_POW(0x04, 0x03) ]

]

TV011B\_TV4\_SR : [4][5]GF28

TV011B\_TV4\_SR = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ],

 [ 0x1A, 0x1E, 0x0A, 0x9D, 0x44 ],

 [ 0x22, 0xE9, 0x73, 0x05, 0x34 ],

 [ 0x4C, 0x76, 0xA0, 0x77, 0x67 ]

]

TV011B\_TV4\_SPLITS : [4][5]GF28

TV011B\_TV4\_SPLITS = [

 [ 0x27, 0xC0, 0x94, 0xBB, 0x54 ],

 [ 0xB9, 0x69, 0xF9, 0xF4, 0x0E ],

 [ 0x7E, 0xC7, 0xCD, 0x32, 0x50 ],

 [ 0xAB, 0xAF, 0x81, 0x82, 0x8D ]

]

/\* test split \*/

property TV011B\_TV4\_split\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV4\_P, TV011B\_TV4\_SR) == TV011B\_TV4\_SPLITS ] == ~zero

TV011B\_TV4\_1\_2\_3\_4\_R : [1][4]GF28

TV011B\_TV4\_1\_2\_3\_4\_R = [

 [ GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x01, 0x03)), GF\_DIV(0x04, GF\_ADD(0x01, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x02, 0x03)), GF\_DIV(0x04, GF\_ADD(0x02, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x03)), GF\_DIV(0x02, GF\_ADD(0x02, 0x03)), GF\_DIV(0x04, GF\_ADD(0x03, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x04)), GF\_DIV(0x02, GF\_ADD(0x02, 0x04)), GF\_DIV(0x03, GF\_ADD(0x03, 0x04))]

 ]

]

TV011B\_TV4\_1\_2\_3\_4\_SPLITS : [4][5]GF28

TV011B\_TV4\_1\_2\_3\_4\_SPLITS = [

 [ 0x27, 0xC0, 0x94, 0xBB, 0x54 ],

 [ 0xB9, 0x69, 0xF9, 0xF4, 0x0E ],

 [ 0x7E, 0xC7, 0xCD, 0x32, 0x50 ],

 [ 0xAB, 0xAF, 0x81, 0x82, 0x8D ]

]

TV011B\_TV4\_SECRET : [1][5]GF28

TV011B\_TV4\_SECRET = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ]

]

/\* test recombines \*/

property TV011B\_TV4\_recombine\_1\_2\_3\_4\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV4\_1\_2\_3\_4\_R, TV011B\_TV4\_1\_2\_3\_4\_SPLITS) == TV011B\_TV4\_SECRET ] == ~zero

/\*

 \* Test vector TV011B\_5

 \* secret = 54 65 73 74 20 44 61 74 61

 \* random = 7F B4 E8 58 1E B7 5D C9 45

 \*

 \* l = 9

 \* m = 2

 \* n = 9

 \*

 \* split1 = 2B D1 9B 2C 3E F3 3C BD 24

 \* split2 = AA 16 B8 C4 1C 31 DB FD EB

 \* split3 = D5 A2 50 9C 02 86 86 34 AE

 \* split4 = B3 83 FE 0F 58 AE 0E 7D 6E

 \* split5 = CC 37 16 57 46 19 53 B4 2B

 \* split6 = 4D F0 35 BF 64 DB B4 F4 E4

 \* split7 = 32 44 DD E7 7A 6C E9 3D A1

 \* split8 = 81 B2 72 82 D0 8B BF 66 7F

 \* split9 = FE 06 9A DA CE 3C E2 AF 3A

 \*/

TV011B\_TV5\_P : [9][2]GF28

TV011B\_TV5\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01) ],

 [ GF\_POW(0x05, 0x00), GF\_POW(0x05, 0x01) ],

 [ GF\_POW(0x06, 0x00), GF\_POW(0x06, 0x01) ],

 [ GF\_POW(0x07, 0x00), GF\_POW(0x07, 0x01) ],

 [ GF\_POW(0x08, 0x00), GF\_POW(0x08, 0x01) ],

 [ GF\_POW(0x09, 0x00), GF\_POW(0x09, 0x01) ]

]

TV011B\_TV5\_SR : [2][9]GF28

TV011B\_TV5\_SR = [

 [ 0x54, 0x65, 0x73, 0x74, 0x20, 0x44, 0x61, 0x74, 0x61 ],

 [ 0x7F, 0xB4, 0xE8, 0x58, 0x1E, 0xB7, 0x5D, 0xC9, 0x45 ]

]

TV011B\_TV5\_SPLITS : [9][9]GF28

TV011B\_TV5\_SPLITS = [

 [ 0x2B, 0xD1, 0x9B, 0x2C, 0x3E, 0xF3, 0x3C, 0xBD, 0x24 ],

 [ 0xAA, 0x16, 0xB8, 0xC4, 0x1C, 0x31, 0xDB, 0xFD, 0xEB ],

 [ 0xD5, 0xA2, 0x50, 0x9C, 0x02, 0x86, 0x86, 0x34, 0xAE ],

 [ 0xB3, 0x83, 0xFE, 0x0F, 0x58, 0xAE, 0x0E, 0x7D, 0x6E ],

 [ 0xCC, 0x37, 0x16, 0x57, 0x46, 0x19, 0x53, 0xB4, 0x2B ],

 [ 0x4D, 0xF0, 0x35, 0xBF, 0x64, 0xDB, 0xB4, 0xF4, 0xE4 ],

 [ 0x32, 0x44, 0xDD, 0xE7, 0x7A, 0x6C, 0xE9, 0x3D, 0xA1 ],

 [ 0x81, 0xB2, 0x72, 0x82, 0xD0, 0x8B, 0xBF, 0x66, 0x7F ],

 [ 0xFE, 0x06, 0x9A, 0xDA, 0xCE, 0x3C, 0xE2, 0xAF, 0x3A ]

]

/\* test split \*/

property TV011B\_TV5\_split\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV5\_P, TV011B\_TV5\_SR) == TV011B\_TV5\_SPLITS ] == ~zero

TV011B\_TV5\_1\_2\_R : [1][2]GF28

TV011B\_TV5\_1\_2\_R = [

 [ GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x01, GF\_ADD(0x01, 0x02)) ]

]

TV011B\_TV5\_8\_9\_R : [1][2]GF28

TV011B\_TV5\_8\_9\_R = [

 [ GF\_DIV(0x09, GF\_ADD(0x08, 0x09)), GF\_DIV(0x08, GF\_ADD(0x08, 0x09)) ]

]

TV011B\_TV5\_1\_2\_SPLITS : [2][9]GF28

TV011B\_TV5\_1\_2\_SPLITS = [

 [ 0x2B, 0xD1, 0x9B, 0x2C, 0x3E, 0xF3, 0x3C, 0xBD, 0x24 ],

 [ 0xAA, 0x16, 0xB8, 0xC4, 0x1C, 0x31, 0xDB, 0xFD, 0xEB ]

]

TV011B\_TV5\_8\_9\_SPLITS : [2][9]GF28

TV011B\_TV5\_8\_9\_SPLITS = [

 [ 0x81, 0xB2, 0x72, 0x82, 0xD0, 0x8B, 0xBF, 0x66, 0x7F ],

 [ 0xFE, 0x06, 0x9A, 0xDA, 0xCE, 0x3C, 0xE2, 0xAF, 0x3A ]

]

TV011B\_TV5\_SECRET : [1][9]GF28

TV011B\_TV5\_SECRET = [

 [ 0x54, 0x65, 0x73, 0x74, 0x20, 0x44, 0x61, 0x74, 0x61 ]

]

/\* test recombines \*/

property TV011B\_TV5\_recombine\_1\_2\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV5\_1\_2\_R, TV011B\_TV5\_1\_2\_SPLITS) == TV011B\_TV5\_SECRET ] == ~zero

property TV011B\_TV5\_recombine\_8\_9\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV5\_8\_9\_R, TV011B\_TV5\_8\_9\_SPLITS) == TV011B\_TV5\_SECRET ] == ~zero

/\*

 \* Test vector TV011B\_6

 \* secret = 01 02 03 04 05 06 07 08 09 0A 0B 0C 0D 0E 0F

 \* random = EC 96 74 05 40 B3 E1 FC 9A 91 4F 6E 5F 7C CA 51 DB 72 32 02 C9 B8 81 00 4F 66 A2 80 71 97

 \*

 \* l = 15

 \* m = 3

 \* n = 5

 \*

 \* split1 = 7B 73 F0 19 0E 27 24 93 A0 3A 7A 8D 24 2C E9

 \* split2 = AC FE 79 00 58 3B 52 D8 77 66 54 15 10 67 87

 \* split3 = D6 8F 8A 1D 53 1A 71 43 DE 56 25 94 39 45 61

 \* split4 = 3F 99 DD F4 88 9B E1 6A 29 E2 77 3E 10 68 63

 \* split5 = 45 E8 2E E9 83 BA C2 F1 80 D2 06 BF 39 4A 85

 \*/

TV011B\_TV6\_P : [5][3]GF28

TV011B\_TV6\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01), GF\_POW(0x01, 0x02) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01), GF\_POW(0x02, 0x02) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01), GF\_POW(0x03, 0x02) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01), GF\_POW(0x04, 0x02) ],

 [ GF\_POW(0x05, 0x00), GF\_POW(0x05, 0x01), GF\_POW(0x05, 0x02) ]

]

TV011B\_TV6\_SR : [3][15]GF28

TV011B\_TV6\_SR = [

 [ 0x01, 0x02, 0x03, 0x04, 0x05, 0x06, 0x07, 0x08, 0x09, 0x0A, 0x0B, 0x0C, 0x0D, 0x0E, 0x0F ],

 [ 0xEC, 0x74, 0x40, 0xE1, 0x9A, 0x4F, 0x5F, 0xCA, 0xDB, 0x32, 0xC9, 0x81, 0x4F, 0xA2, 0x71 ],

 [ 0x96, 0x05, 0xB3, 0xFC, 0x91, 0x6E, 0x7C, 0x51, 0x72, 0x02, 0xB8, 0x00, 0x66, 0x80, 0x97 ]

]

TV011B\_TV6\_SPLITS : [5][15]GF28

TV011B\_TV6\_SPLITS = [

 [ 0x7B, 0x73, 0xF0, 0x19, 0x0E, 0x27, 0x24, 0x93, 0xA0, 0x3A, 0x7A, 0x8D, 0x24, 0x2C, 0xE9 ],

 [ 0xAC, 0xFE, 0x79, 0x00, 0x58, 0x3B, 0x52, 0xD8, 0x77, 0x66, 0x54, 0x15, 0x10, 0x67, 0x87 ],

 [ 0xD6, 0x8F, 0x8A, 0x1D, 0x53, 0x1A, 0x71, 0x43, 0xDE, 0x56, 0x25, 0x94, 0x39, 0x45, 0x61 ],

 [ 0x3F, 0x99, 0xDD, 0xF4, 0x88, 0x9B, 0xE1, 0x6A, 0x29, 0xE2, 0x77, 0x3E, 0x10, 0x68, 0x63 ],

 [ 0x45, 0xE8, 0x2E, 0xE9, 0x83, 0xBA, 0xC2, 0xF1, 0x80, 0xD2, 0x06, 0xBF, 0x39, 0x4A, 0x85 ]

]

/\* test split \*/

property TV011B\_TV6\_split\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV6\_P, TV011B\_TV6\_SR) == TV011B\_TV6\_SPLITS ] == ~zero

TV011B\_TV6\_1\_2\_3\_R : [1][3]GF28

TV011B\_TV6\_1\_2\_3\_R = [

 [ GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x01, 0x03))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x02, 0x03))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x03)), GF\_DIV(0x02, GF\_ADD(0x02, 0x03))]

 ]

]

TV011B\_TV6\_2\_3\_4\_R : [1][3]GF28

TV011B\_TV6\_2\_3\_4\_R = [

 [ GF\_PROD[GF\_DIV(0x03, GF\_ADD(0x02, 0x03)), GF\_DIV(0x04, GF\_ADD(0x02, 0x04))],

 GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x02, 0x03)), GF\_DIV(0x04, GF\_ADD(0x03, 0x04))],

 GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x02, 0x04)), GF\_DIV(0x03, GF\_ADD(0x03, 0x04))]

 ]

]

TV011B\_TV6\_1\_2\_3\_SPLITS : [3][15]GF28

TV011B\_TV6\_1\_2\_3\_SPLITS = [

 [ 0x7B, 0x73, 0xF0, 0x19, 0x0E, 0x27, 0x24, 0x93, 0xA0, 0x3A, 0x7A, 0x8D, 0x24, 0x2C, 0xE9 ],

 [ 0xAC, 0xFE, 0x79, 0x00, 0x58, 0x3B, 0x52, 0xD8, 0x77, 0x66, 0x54, 0x15, 0x10, 0x67, 0x87 ],

 [ 0xD6, 0x8F, 0x8A, 0x1D, 0x53, 0x1A, 0x71, 0x43, 0xDE, 0x56, 0x25, 0x94, 0x39, 0x45, 0x61 ]

]

TV011B\_TV6\_2\_3\_4\_SPLITS : [3][15]GF28

TV011B\_TV6\_2\_3\_4\_SPLITS = [

 [ 0xAC, 0xFE, 0x79, 0x00, 0x58, 0x3B, 0x52, 0xD8, 0x77, 0x66, 0x54, 0x15, 0x10, 0x67, 0x87 ],

 [ 0xD6, 0x8F, 0x8A, 0x1D, 0x53, 0x1A, 0x71, 0x43, 0xDE, 0x56, 0x25, 0x94, 0x39, 0x45, 0x61 ],

 [ 0x3F, 0x99, 0xDD, 0xF4, 0x88, 0x9B, 0xE1, 0x6A, 0x29, 0xE2, 0x77, 0x3E, 0x10, 0x68, 0x63 ]

]

TV011B\_TV6\_SECRET : [1][15]GF28

TV011B\_TV6\_SECRET = [

 [ 0x01, 0x02, 0x03, 0x04, 0x05, 0x06, 0x07, 0x08, 0x09, 0x0A, 0x0B, 0x0C, 0x0D, 0x0E, 0x0F ]

]

/\* test recombines \*/

property TV011B\_TV6\_recombine\_1\_2\_3\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV6\_1\_2\_3\_R, TV011B\_TV6\_1\_2\_3\_SPLITS) == TV011B\_TV6\_SECRET ] == ~zero

property TV011B\_TV6\_recombine\_2\_3\_4\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV6\_2\_3\_4\_R, TV011B\_TV6\_2\_3\_4\_SPLITS) == TV011B\_TV6\_SECRET ] == ~zero

* + 1. “Native” GF(256)-based implementation

module secret\_share\_011B\_gf28 where

type GF28 = [8]

irreducible = <| x^^8 + x^^4 + x^^3 + x + 1 |>

GF\_ADD : (GF28, GF28) -> GF28

GF\_ADD (x, y) = x ^ y

GF\_SUB : (GF28, GF28) -> GF28

GF\_SUB (x, y) = x ^ y

GF\_MUL : (GF28, GF28) -> GF28

GF\_MUL (x, y) = pmod(pmult x y) irreducible

GF\_POW : (GF28, [8]) -> GF28

GF\_POW (n, k) = pow k

 where sq x = GF\_MUL (x, x)

 odd x = x ! 0

 pow i = if i == 0 then 1

 else if odd i

 then GF\_MUL(n, sq (pow (i >> 1)))

 else sq (pow (i >> 1))

GF\_INV : GF28 -> GF28

GF\_INV x = GF\_POW (x, 254)

GF\_DIV : (GF28, GF28) -> GF28

GF\_DIV (x, y) = div x

 where inv yi = GF\_INV (yi)

 div i = if x == 0 then 0

 else GF\_MUL(x, inv y)

GF\_SUM : {n} (fin n) => [n]GF28 -> GF28

GF\_SUM ps = accum ! 0

 where accum = [zero] # [ GF\_ADD(p, a) | p <- ps | a <- accum ]

GF\_PROD : {n} (fin n) => [n]GF28 -> GF28

GF\_PROD ps = accum ! 0

 where accum = [1] # [ GF\_MUL(p, a) | p <- ps | a <- accum ]

GF\_DOT\_PROD : {n} (fin n) => ([n]GF28, [n]GF28) -> GF28

GF\_DOT\_PROD (xs, ys) = GF\_SUM [ GF\_MUL (x, y) | x <- xs

 | y <- ys

 ]

GF\_VEC\_MUL : {n, m} (fin n) => ([n]GF28, [m][n]GF28) -> [m]GF28

GF\_VEC\_MUL (v, ms) = [ GF\_DOT\_PROD(v, m) | m <- ms ]

GF\_MAT\_MUL : {n, m, k} (fin m) => ([n][m]GF28, [m][k]GF28) -> [n][k]GF28

GF\_MAT\_MUL (xss, yss) = [ GF\_VEC\_MUL(xs, yss') | xs <- xss ]

 where yss' = transpose yss

// Test test vectors for Polynomial 1 (x^^8 + x^^4 + x^^3 + x + 1)

/\*

 \* Test vector TV011B\_1

 \* secret = 74 65 73 74 00

 \* random = A8 7B 34 91 B5

 \*

 \* l = 5

 \* m = 2

 \* n = 2

 \*

 \* split1 = DC 1E 47 E5 B5

 \* split2 = 3F 93 1B 4D 71

 \*/

TV011B\_TV1\_P : [2][2]GF28

TV011B\_TV1\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01) ]

]

TV011B\_TV1\_SR : [2][5]GF28

TV011B\_TV1\_SR = [

 [ 0x74, 0x65, 0x73, 0x74, 0x00 ],

 [ 0xA8, 0x7B, 0x34, 0x91, 0xB5 ]

]

TV011B\_TV1\_SPLITS : [2][5]GF28

TV011B\_TV1\_SPLITS = [

 [ 0xDC, 0x1E, 0x47, 0xE5, 0xB5 ],

 [ 0x3F, 0x93, 0x1B, 0x4D, 0x71 ]

]

/\* test split \*/

property TV011B\_TV1\_split\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV1\_P, TV011B\_TV1\_SR) == TV011B\_TV1\_SPLITS ] == ~zero

TV011B\_TV1\_1\_2\_R : [1][2]GF28

TV011B\_TV1\_1\_2\_R = [

 [ GF\_DIV(0x02, GF\_ADD(0x02, 0x01)), GF\_DIV(0x01, GF\_ADD(0x01, 0x02)) ]

]

TV011B\_TV1\_1\_2\_SPLITS : [2][5]GF28

TV011B\_TV1\_1\_2\_SPLITS = [

 [ 0xDC, 0x1E, 0x47, 0xE5, 0xB5 ],

 [ 0x3F, 0x93, 0x1B, 0x4D, 0x71 ]

]

TV011B\_TV1\_SECRET : [1][5]GF28

TV011B\_TV1\_SECRET = [

 [ 0x74, 0x65, 0x73, 0x74, 0x00 ]

]

/\* test recombine \*/

property TV011B\_TV1\_recombine\_1\_2\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV1\_1\_2\_R, TV011B\_TV1\_1\_2\_SPLITS) == TV011B\_TV1\_SECRET ] == ~zero

/\*

 \* Test vector TV011B\_2

 \* secret = 53 41 4D 54 43

 \* random = 39 5D 39 6C 87

 \*

 \* l = 5

 \* m = 2

 \* n = 4

 \*

 \* split1 = 6A 1C 74 38 C4

 \* split2 = 21 FB 3F 8C 56

 \* split3 = 18 A6 06 E0 D1

 \* split4 = B7 2E A9 FF 69

 \*/

TV011B\_TV2\_P : [4][2]GF28

TV011B\_TV2\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01) ]

]

TV011B\_TV2\_SR : [2][5]GF28

TV011B\_TV2\_SR = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ],

 [ 0x39, 0x5D, 0x39, 0x6C, 0x87 ]

]

TV011B\_TV2\_SPLITS : [4][5]GF28

TV011B\_TV2\_SPLITS = [

 [ 0x6A, 0x1C, 0x74, 0x38, 0xC4 ],

 [ 0x21, 0xFB, 0x3F, 0x8C, 0x56 ],

 [ 0x18, 0xA6, 0x06, 0xE0, 0xD1 ],

 [ 0xB7, 0x2E, 0xA9, 0xFF, 0x69 ]

]

/\* test split \*/

property TV011B\_TV2\_split\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV2\_P, TV011B\_TV2\_SR) == TV011B\_TV2\_SPLITS ] == ~zero

TV011B\_TV2\_1\_2\_R : [1][2]GF28

TV011B\_TV2\_1\_2\_R = [

 [ GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x01, GF\_ADD(0x01, 0x02)) ]

]

TV011B\_TV2\_1\_4\_R : [1][2]GF28

TV011B\_TV2\_1\_4\_R = [

 [ GF\_DIV(0x04, GF\_ADD(0x01, 0x04)), GF\_DIV(0x01, GF\_ADD(0x01, 0x04)) ]

]

TV011B\_TV2\_3\_4\_R : [1][2]GF28

TV011B\_TV2\_3\_4\_R = [

 [ GF\_DIV(0x04, GF\_ADD(0x03, 0x04)), GF\_DIV(0x03, GF\_ADD(0x03, 0x04)) ]

]

TV011B\_TV2\_1\_2\_SPLITS : [2][5]GF28

TV011B\_TV2\_1\_2\_SPLITS = [

 [ 0x6A, 0x1C, 0x74, 0x38, 0xC4 ],

 [ 0x21, 0xFB, 0x3F, 0x8C, 0x56 ]

]

TV011B\_TV2\_1\_4\_SPLITS : [2][5]GF28

TV011B\_TV2\_1\_4\_SPLITS = [

 [ 0x6A, 0x1C, 0x74, 0x38, 0xC4 ],

 [ 0xB7, 0x2E, 0xA9, 0xFF, 0x69 ]

]

TV011B\_TV2\_3\_4\_SPLITS : [2][5]GF28

TV011B\_TV2\_3\_4\_SPLITS = [

 [ 0x18, 0xA6, 0x06, 0xE0, 0xD1 ],

 [ 0xB7, 0x2E, 0xA9, 0xFF, 0x69 ]

]

TV011B\_TV2\_SECRET : [1][5]GF28

TV011B\_TV2\_SECRET = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ]

]

/\* test recombines \*/

property TV011B\_TV2\_recombine\_1\_2\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV2\_1\_2\_R, TV011B\_TV2\_1\_2\_SPLITS) == TV011B\_TV2\_SECRET ] == ~zero

property TV011B\_TV2\_recombine\_1\_4\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV2\_1\_4\_R, TV011B\_TV2\_1\_4\_SPLITS) == TV011B\_TV2\_SECRET ] == ~zero

property TV011B\_TV2\_recombine\_3\_4\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV2\_3\_4\_R, TV011B\_TV2\_3\_4\_SPLITS) == TV011B\_TV2\_SECRET ] == ~zero

/\*

 \* Test vector TV011B\_3

 \* secret = 53 41 4D 54 43

 \* random = 27 3A 1A 28 AB 99 79 BC 06 37

 \*

 \* l = 5

 \* m = 3

 \* n = 4

 \*

 \* split1 = 4E 73 7F 91 72

 \* split2 = F5 D5 52 60 93

 \* split3 = E8 E7 60 A5 A2

 \* split4 = 42 9F 84 9E 06

 \*/

TV011B\_TV3\_P : [4][3]GF28

TV011B\_TV3\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01), GF\_POW(0x01, 0x02) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01), GF\_POW(0x02, 0x02) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01), GF\_POW(0x03, 0x02) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01), GF\_POW(0x04, 0x02) ]

]

TV011B\_TV3\_SR : [3][5]GF28

TV011B\_TV3\_SR = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ],

 [ 0x27, 0x1A, 0xAB, 0x79, 0x06 ],

 [ 0x3A, 0x28, 0x99, 0xBC, 0x37 ]

]

TV011B\_TV3\_SPLITS : [4][5]GF28

TV011B\_TV3\_SPLITS = [

 [ 0x4E, 0x73, 0x7F, 0x91, 0x72 ],

 [ 0xF5, 0xD5, 0x52, 0x60, 0x93 ],

 [ 0xE8, 0xE7, 0x60, 0xA5, 0xA2 ],

 [ 0x42, 0x9F, 0x84, 0x9E, 0x06 ]

]

/\* test split \*/

property TV011B\_TV3\_split\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV3\_P, TV011B\_TV3\_SR) == TV011B\_TV3\_SPLITS ] == ~zero

TV011B\_TV3\_1\_2\_3\_R : [1][3]GF28

TV011B\_TV3\_1\_2\_3\_R = [

 [ GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x01, 0x03))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x02, 0x03))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x03)), GF\_DIV(0x02, GF\_ADD(0x02, 0x03))]

 ]

]

TV011B\_TV3\_1\_2\_4\_R : [1][3]GF28

TV011B\_TV3\_1\_2\_4\_R = [

 [ GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x04, GF\_ADD(0x01, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x02)), GF\_DIV(0x04, GF\_ADD(0x02, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x04)), GF\_DIV(0x02, GF\_ADD(0x02, 0x04))]

 ]

]

TV011B\_TV3\_1\_3\_4\_R : [1][3]GF28

TV011B\_TV3\_1\_3\_4\_R = [

 [ GF\_PROD[GF\_DIV(0x03, GF\_ADD(0x01, 0x03)), GF\_DIV(0x04, GF\_ADD(0x01, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x03)), GF\_DIV(0x04, GF\_ADD(0x03, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x04)), GF\_DIV(0x03, GF\_ADD(0x03, 0x04))]

 ]

]

TV011B\_TV3\_1\_2\_3\_SPLITS : [3][5]GF28

TV011B\_TV3\_1\_2\_3\_SPLITS = [

 [ 0x4E, 0x73, 0x7F, 0x91, 0x72 ],

 [ 0xF5, 0xD5, 0x52, 0x60, 0x93 ],

 [ 0xE8, 0xE7, 0x60, 0xA5, 0xA2 ]

]

TV011B\_TV3\_1\_2\_4\_SPLITS : [3][5]GF28

TV011B\_TV3\_1\_2\_4\_SPLITS = [

 [ 0x4E, 0x73, 0x7F, 0x91, 0x72 ],

 [ 0xF5, 0xD5, 0x52, 0x60, 0x93 ],

 [ 0x42, 0x9F, 0x84, 0x9E, 0x06 ]

]

TV011B\_TV3\_1\_3\_4\_SPLITS : [3][5]GF28

TV011B\_TV3\_1\_3\_4\_SPLITS = [

 [ 0x4E, 0x73, 0x7F, 0x91, 0x72 ],

 [ 0xE8, 0xE7, 0x60, 0xA5, 0xA2 ],

 [ 0x42, 0x9F, 0x84, 0x9E, 0x06 ]

]

TV011B\_TV3\_SECRET : [1][5]GF28

TV011B\_TV3\_SECRET = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ]

]

/\* test recombines \*/

property TV011B\_TV3\_recombine\_1\_2\_3\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV3\_1\_2\_3\_R, TV011B\_TV3\_1\_2\_3\_SPLITS) == TV011B\_TV3\_SECRET ] == ~zero

property TV011B\_TV3\_recombine\_1\_2\_4\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV3\_1\_2\_4\_R, TV011B\_TV3\_1\_2\_4\_SPLITS) == TV011B\_TV3\_SECRET ] == ~zero

property TV011B\_TV3\_recombine\_1\_3\_4\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV3\_1\_3\_4\_R, TV011B\_TV3\_1\_3\_4\_SPLITS) == TV011B\_TV3\_SECRET ] == ~zero

/\*

 \* Test vector TV011B\_4

 \* secret = 53 41 4D 54 43

 \* random = 1A 22 4C 1E E9 76 0A 73 A0 9D 05 77 44 34 67

 \*

 \* l = 5

 \* m = 4

 \* n = 4

 \*

 \* split1 = 27 C0 94 BB 54

 \* split2 = B9 69 F9 F4 0E

 \* split3 = 7E C7 CD 32 50

 \* split4 = AB AF 81 82 8D

 \*/

TV011B\_TV4\_P : [4][4]GF28

TV011B\_TV4\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01), GF\_POW(0x01, 0x02), GF\_POW(0x01, 0x03) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01), GF\_POW(0x02, 0x02), GF\_POW(0x02, 0x03) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01), GF\_POW(0x03, 0x02), GF\_POW(0x03, 0x03) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01), GF\_POW(0x04, 0x02), GF\_POW(0x04, 0x03) ]

]

TV011B\_TV4\_SR : [4][5]GF28

TV011B\_TV4\_SR = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ],

 [ 0x1A, 0x1E, 0x0A, 0x9D, 0x44 ],

 [ 0x22, 0xE9, 0x73, 0x05, 0x34 ],

 [ 0x4C, 0x76, 0xA0, 0x77, 0x67 ]

]

TV011B\_TV4\_SPLITS : [4][5]GF28

TV011B\_TV4\_SPLITS = [

 [ 0x27, 0xC0, 0x94, 0xBB, 0x54 ],

 [ 0xB9, 0x69, 0xF9, 0xF4, 0x0E ],

 [ 0x7E, 0xC7, 0xCD, 0x32, 0x50 ],

 [ 0xAB, 0xAF, 0x81, 0x82, 0x8D ]

]

/\* test split \*/

property TV011B\_TV4\_split\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV4\_P, TV011B\_TV4\_SR) == TV011B\_TV4\_SPLITS ] == ~zero

TV011B\_TV4\_1\_2\_3\_4\_R : [1][4]GF28

TV011B\_TV4\_1\_2\_3\_4\_R = [

 [ GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x01, 0x03)), GF\_DIV(0x04, GF\_ADD(0x01, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x02, 0x03)), GF\_DIV(0x04, GF\_ADD(0x02, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x03)), GF\_DIV(0x02, GF\_ADD(0x02, 0x03)), GF\_DIV(0x04, GF\_ADD(0x03, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x04)), GF\_DIV(0x02, GF\_ADD(0x02, 0x04)), GF\_DIV(0x03, GF\_ADD(0x03, 0x04))]

 ]

]

TV011B\_TV4\_1\_2\_3\_4\_SPLITS : [4][5]GF28

TV011B\_TV4\_1\_2\_3\_4\_SPLITS = [

 [ 0x27, 0xC0, 0x94, 0xBB, 0x54 ],

 [ 0xB9, 0x69, 0xF9, 0xF4, 0x0E ],

 [ 0x7E, 0xC7, 0xCD, 0x32, 0x50 ],

 [ 0xAB, 0xAF, 0x81, 0x82, 0x8D ]

]

TV011B\_TV4\_SECRET : [1][5]GF28

TV011B\_TV4\_SECRET = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ]

]

/\* test recombines \*/

property TV011B\_TV4\_recombine\_1\_2\_3\_4\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV4\_1\_2\_3\_4\_R, TV011B\_TV4\_1\_2\_3\_4\_SPLITS) == TV011B\_TV4\_SECRET ] == ~zero

/\*

 \* Test vector TV011B\_5

 \* secret = 54 65 73 74 20 44 61 74 61

 \* random = 7F B4 E8 58 1E B7 5D C9 45

 \*

 \* l = 9

 \* m = 2

 \* n = 9

 \*

 \* split1 = 2B D1 9B 2C 3E F3 3C BD 24

 \* split2 = AA 16 B8 C4 1C 31 DB FD EB

 \* split3 = D5 A2 50 9C 02 86 86 34 AE

 \* split4 = B3 83 FE 0F 58 AE 0E 7D 6E

 \* split5 = CC 37 16 57 46 19 53 B4 2B

 \* split6 = 4D F0 35 BF 64 DB B4 F4 E4

 \* split7 = 32 44 DD E7 7A 6C E9 3D A1

 \* split8 = 81 B2 72 82 D0 8B BF 66 7F

 \* split9 = FE 06 9A DA CE 3C E2 AF 3A

 \*/

TV011B\_TV5\_P : [9][2]GF28

TV011B\_TV5\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01) ],

 [ GF\_POW(0x05, 0x00), GF\_POW(0x05, 0x01) ],

 [ GF\_POW(0x06, 0x00), GF\_POW(0x06, 0x01) ],

 [ GF\_POW(0x07, 0x00), GF\_POW(0x07, 0x01) ],

 [ GF\_POW(0x08, 0x00), GF\_POW(0x08, 0x01) ],

 [ GF\_POW(0x09, 0x00), GF\_POW(0x09, 0x01) ]

]

TV011B\_TV5\_SR : [2][9]GF28

TV011B\_TV5\_SR = [

 [ 0x54, 0x65, 0x73, 0x74, 0x20, 0x44, 0x61, 0x74, 0x61 ],

 [ 0x7F, 0xB4, 0xE8, 0x58, 0x1E, 0xB7, 0x5D, 0xC9, 0x45 ]

]

TV011B\_TV5\_SPLITS : [9][9]GF28

TV011B\_TV5\_SPLITS = [

 [ 0x2B, 0xD1, 0x9B, 0x2C, 0x3E, 0xF3, 0x3C, 0xBD, 0x24 ],

 [ 0xAA, 0x16, 0xB8, 0xC4, 0x1C, 0x31, 0xDB, 0xFD, 0xEB ],

 [ 0xD5, 0xA2, 0x50, 0x9C, 0x02, 0x86, 0x86, 0x34, 0xAE ],

 [ 0xB3, 0x83, 0xFE, 0x0F, 0x58, 0xAE, 0x0E, 0x7D, 0x6E ],

 [ 0xCC, 0x37, 0x16, 0x57, 0x46, 0x19, 0x53, 0xB4, 0x2B ],

 [ 0x4D, 0xF0, 0x35, 0xBF, 0x64, 0xDB, 0xB4, 0xF4, 0xE4 ],

 [ 0x32, 0x44, 0xDD, 0xE7, 0x7A, 0x6C, 0xE9, 0x3D, 0xA1 ],

 [ 0x81, 0xB2, 0x72, 0x82, 0xD0, 0x8B, 0xBF, 0x66, 0x7F ],

 [ 0xFE, 0x06, 0x9A, 0xDA, 0xCE, 0x3C, 0xE2, 0xAF, 0x3A ]

]

/\* test split \*/

property TV011B\_TV5\_split\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV5\_P, TV011B\_TV5\_SR) == TV011B\_TV5\_SPLITS ] == ~zero

TV011B\_TV5\_1\_2\_R : [1][2]GF28

TV011B\_TV5\_1\_2\_R = [

 [ GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x01, GF\_ADD(0x01, 0x02)) ]

]

TV011B\_TV5\_8\_9\_R : [1][2]GF28

TV011B\_TV5\_8\_9\_R = [

 [ GF\_DIV(0x09, GF\_ADD(0x08, 0x09)), GF\_DIV(0x08, GF\_ADD(0x08, 0x09)) ]

]

TV011B\_TV5\_1\_2\_SPLITS : [2][9]GF28

TV011B\_TV5\_1\_2\_SPLITS = [

 [ 0x2B, 0xD1, 0x9B, 0x2C, 0x3E, 0xF3, 0x3C, 0xBD, 0x24 ],

 [ 0xAA, 0x16, 0xB8, 0xC4, 0x1C, 0x31, 0xDB, 0xFD, 0xEB ]

]

TV011B\_TV5\_8\_9\_SPLITS : [2][9]GF28

TV011B\_TV5\_8\_9\_SPLITS = [

 [ 0x81, 0xB2, 0x72, 0x82, 0xD0, 0x8B, 0xBF, 0x66, 0x7F ],

 [ 0xFE, 0x06, 0x9A, 0xDA, 0xCE, 0x3C, 0xE2, 0xAF, 0x3A ]

]

TV011B\_TV5\_SECRET : [1][9]GF28

TV011B\_TV5\_SECRET = [

 [ 0x54, 0x65, 0x73, 0x74, 0x20, 0x44, 0x61, 0x74, 0x61 ]

]

/\* test recombines \*/

property TV011B\_TV5\_recombine\_1\_2\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV5\_1\_2\_R, TV011B\_TV5\_1\_2\_SPLITS) == TV011B\_TV5\_SECRET ] == ~zero

property TV011B\_TV5\_recombine\_8\_9\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV5\_8\_9\_R, TV011B\_TV5\_8\_9\_SPLITS) == TV011B\_TV5\_SECRET ] == ~zero

/\*

 \* Test vector TV011B\_6

 \* secret = 01 02 03 04 05 06 07 08 09 0A 0B 0C 0D 0E 0F

 \* random = EC 96 74 05 40 B3 E1 FC 9A 91 4F 6E 5F 7C CA 51 DB 72 32 02 C9 B8 81 00 4F 66 A2 80 71 97

 \*

 \* l = 15

 \* m = 3

 \* n = 5

 \*

 \* split1 = 7B 73 F0 19 0E 27 24 93 A0 3A 7A 8D 24 2C E9

 \* split2 = AC FE 79 00 58 3B 52 D8 77 66 54 15 10 67 87

 \* split3 = D6 8F 8A 1D 53 1A 71 43 DE 56 25 94 39 45 61

 \* split4 = 3F 99 DD F4 88 9B E1 6A 29 E2 77 3E 10 68 63

 \* split5 = 45 E8 2E E9 83 BA C2 F1 80 D2 06 BF 39 4A 85

 \*/

TV011B\_TV6\_P : [5][3]GF28

TV011B\_TV6\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01), GF\_POW(0x01, 0x02) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01), GF\_POW(0x02, 0x02) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01), GF\_POW(0x03, 0x02) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01), GF\_POW(0x04, 0x02) ],

 [ GF\_POW(0x05, 0x00), GF\_POW(0x05, 0x01), GF\_POW(0x05, 0x02) ]

]

TV011B\_TV6\_SR : [3][15]GF28

TV011B\_TV6\_SR = [

 [ 0x01, 0x02, 0x03, 0x04, 0x05, 0x06, 0x07, 0x08, 0x09, 0x0A, 0x0B, 0x0C, 0x0D, 0x0E, 0x0F ],

 [ 0xEC, 0x74, 0x40, 0xE1, 0x9A, 0x4F, 0x5F, 0xCA, 0xDB, 0x32, 0xC9, 0x81, 0x4F, 0xA2, 0x71 ],

 [ 0x96, 0x05, 0xB3, 0xFC, 0x91, 0x6E, 0x7C, 0x51, 0x72, 0x02, 0xB8, 0x00, 0x66, 0x80, 0x97 ]

]

TV011B\_TV6\_SPLITS : [5][15]GF28

TV011B\_TV6\_SPLITS = [

 [ 0x7B, 0x73, 0xF0, 0x19, 0x0E, 0x27, 0x24, 0x93, 0xA0, 0x3A, 0x7A, 0x8D, 0x24, 0x2C, 0xE9 ],

 [ 0xAC, 0xFE, 0x79, 0x00, 0x58, 0x3B, 0x52, 0xD8, 0x77, 0x66, 0x54, 0x15, 0x10, 0x67, 0x87 ],

 [ 0xD6, 0x8F, 0x8A, 0x1D, 0x53, 0x1A, 0x71, 0x43, 0xDE, 0x56, 0x25, 0x94, 0x39, 0x45, 0x61 ],

 [ 0x3F, 0x99, 0xDD, 0xF4, 0x88, 0x9B, 0xE1, 0x6A, 0x29, 0xE2, 0x77, 0x3E, 0x10, 0x68, 0x63 ],

 [ 0x45, 0xE8, 0x2E, 0xE9, 0x83, 0xBA, 0xC2, 0xF1, 0x80, 0xD2, 0x06, 0xBF, 0x39, 0x4A, 0x85 ]

]

/\* test split \*/

property TV011B\_TV6\_split\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV6\_P, TV011B\_TV6\_SR) == TV011B\_TV6\_SPLITS ] == ~zero

TV011B\_TV6\_1\_2\_3\_R : [1][3]GF28

TV011B\_TV6\_1\_2\_3\_R = [

 [ GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x01, 0x03))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x02, 0x03))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x03)), GF\_DIV(0x02, GF\_ADD(0x02, 0x03))]

 ]

]

TV011B\_TV6\_2\_3\_4\_R : [1][3]GF28

TV011B\_TV6\_2\_3\_4\_R = [

 [ GF\_PROD[GF\_DIV(0x03, GF\_ADD(0x02, 0x03)), GF\_DIV(0x04, GF\_ADD(0x02, 0x04))],

 GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x02, 0x03)), GF\_DIV(0x04, GF\_ADD(0x03, 0x04))],

 GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x02, 0x04)), GF\_DIV(0x03, GF\_ADD(0x03, 0x04))]

 ]

]

TV011B\_TV6\_1\_2\_3\_SPLITS : [3][15]GF28

TV011B\_TV6\_1\_2\_3\_SPLITS = [

 [ 0x7B, 0x73, 0xF0, 0x19, 0x0E, 0x27, 0x24, 0x93, 0xA0, 0x3A, 0x7A, 0x8D, 0x24, 0x2C, 0xE9 ],

 [ 0xAC, 0xFE, 0x79, 0x00, 0x58, 0x3B, 0x52, 0xD8, 0x77, 0x66, 0x54, 0x15, 0x10, 0x67, 0x87 ],

 [ 0xD6, 0x8F, 0x8A, 0x1D, 0x53, 0x1A, 0x71, 0x43, 0xDE, 0x56, 0x25, 0x94, 0x39, 0x45, 0x61 ]

]

TV011B\_TV6\_2\_3\_4\_SPLITS : [3][15]GF28

TV011B\_TV6\_2\_3\_4\_SPLITS = [

 [ 0xAC, 0xFE, 0x79, 0x00, 0x58, 0x3B, 0x52, 0xD8, 0x77, 0x66, 0x54, 0x15, 0x10, 0x67, 0x87 ],

 [ 0xD6, 0x8F, 0x8A, 0x1D, 0x53, 0x1A, 0x71, 0x43, 0xDE, 0x56, 0x25, 0x94, 0x39, 0x45, 0x61 ],

 [ 0x3F, 0x99, 0xDD, 0xF4, 0x88, 0x9B, 0xE1, 0x6A, 0x29, 0xE2, 0x77, 0x3E, 0x10, 0x68, 0x63 ]

]

TV011B\_TV6\_SECRET : [1][15]GF28

TV011B\_TV6\_SECRET = [

 [ 0x01, 0x02, 0x03, 0x04, 0x05, 0x06, 0x07, 0x08, 0x09, 0x0A, 0x0B, 0x0C, 0x0D, 0x0E, 0x0F ]

]

/\* test recombines \*/

property TV011B\_TV6\_recombine\_1\_2\_3\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV6\_1\_2\_3\_R, TV011B\_TV6\_1\_2\_3\_SPLITS) == TV011B\_TV6\_SECRET ] == ~zero

property TV011B\_TV6\_recombine\_2\_3\_4\_passes\_test = [ GF\_MAT\_MUL(TV011B\_TV6\_2\_3\_4\_R, TV011B\_TV6\_2\_3\_4\_SPLITS) == TV011B\_TV6\_SECRET ] == ~zero

* 1. Polynomial

Cryptol code for creating splits, checking the splits, re-combining splits, and checking the recombinations appears in the following subsections.

* + 1. Table-based implementation

module secret\_share\_011D\_table where

type GF28 = [8]

/\* given an alpha^j, where alpha = mimimum primitive element (x = 2), return j \*/

LOG : [256][8]

LOG = [

 0xff, 0x00, 0x01, 0x19, 0x02, 0x32, 0x1a, 0xc6,

 0x03, 0xdf, 0x33, 0xee, 0x1b, 0x68, 0xc7, 0x4b,

 0x04, 0x64, 0xe0, 0x0e, 0x34, 0x8d, 0xef, 0x81,

 0x1c, 0xc1, 0x69, 0xf8, 0xc8, 0x08, 0x4c, 0x71,

 0x05, 0x8a, 0x65, 0x2f, 0xe1, 0x24, 0x0f, 0x21,

 0x35, 0x93, 0x8e, 0xda, 0xf0, 0x12, 0x82, 0x45,

 0x1d, 0xb5, 0xc2, 0x7d, 0x6a, 0x27, 0xf9, 0xb9,

 0xc9, 0x9a, 0x09, 0x78, 0x4d, 0xe4, 0x72, 0xa6,

 0x06, 0xbf, 0x8b, 0x62, 0x66, 0xdd, 0x30, 0xfd,

 0xe2, 0x98, 0x25, 0xb3, 0x10, 0x91, 0x22, 0x88,

 0x36, 0xd0, 0x94, 0xce, 0x8f, 0x96, 0xdb, 0xbd,

 0xf1, 0xd2, 0x13, 0x5c, 0x83, 0x38, 0x46, 0x40,

 0x1e, 0x42, 0xb6, 0xa3, 0xc3, 0x48, 0x7e, 0x6e,

 0x6b, 0x3a, 0x28, 0x54, 0xfa, 0x85, 0xba, 0x3d,

 0xca, 0x5e, 0x9b, 0x9f, 0x0a, 0x15, 0x79, 0x2b,

 0x4e, 0xd4, 0xe5, 0xac, 0x73, 0xf3, 0xa7, 0x57,

 0x07, 0x70, 0xc0, 0xf7, 0x8c, 0x80, 0x63, 0x0d,

 0x67, 0x4a, 0xde, 0xed, 0x31, 0xc5, 0xfe, 0x18,

 0xe3, 0xa5, 0x99, 0x77, 0x26, 0xb8, 0xb4, 0x7c,

 0x11, 0x44, 0x92, 0xd9, 0x23, 0x20, 0x89, 0x2e,

 0x37, 0x3f, 0xd1, 0x5b, 0x95, 0xbc, 0xcf, 0xcd,

 0x90, 0x87, 0x97, 0xb2, 0xdc, 0xfc, 0xbe, 0x61,

 0xf2, 0x56, 0xd3, 0xab, 0x14, 0x2a, 0x5d, 0x9e,

 0x84, 0x3c, 0x39, 0x53, 0x47, 0x6d, 0x41, 0xa2,

 0x1f, 0x2d, 0x43, 0xd8, 0xb7, 0x7b, 0xa4, 0x76,

 0xc4, 0x17, 0x49, 0xec, 0x7f, 0x0c, 0x6f, 0xf6,

 0x6c, 0xa1, 0x3b, 0x52, 0x29, 0x9d, 0x55, 0xaa,

 0xfb, 0x60, 0x86, 0xb1, 0xbb, 0xcc, 0x3e, 0x5a,

 0xcb, 0x59, 0x5f, 0xb0, 0x9c, 0xa9, 0xa0, 0x51,

 0x0b, 0xf5, 0x16, 0xeb, 0x7a, 0x75, 0x2c, 0xd7,

 0x4f, 0xae, 0xd5, 0xe9, 0xe6, 0xe7, 0xad, 0xe8,

 0x74, 0xd6, 0xf4, 0xea, 0xa8, 0x50, 0x58, 0xaf

]

/\* given a j, return alpha^j, where alpha = mimimum primitive element (x = 2) \*/

EXP : [256][8]

EXP = [

 0x01, 0x02, 0x04, 0x08, 0x10, 0x20, 0x40, 0x80,

 0x1d, 0x3a, 0x74, 0xe8, 0xcd, 0x87, 0x13, 0x26,

 0x4c, 0x98, 0x2d, 0x5a, 0xb4, 0x75, 0xea, 0xc9,

 0x8f, 0x03, 0x06, 0x0c, 0x18, 0x30, 0x60, 0xc0,

 0x9d, 0x27, 0x4e, 0x9c, 0x25, 0x4a, 0x94, 0x35,

 0x6a, 0xd4, 0xb5, 0x77, 0xee, 0xc1, 0x9f, 0x23,

 0x46, 0x8c, 0x05, 0x0a, 0x14, 0x28, 0x50, 0xa0,

 0x5d, 0xba, 0x69, 0xd2, 0xb9, 0x6f, 0xde, 0xa1,

 0x5f, 0xbe, 0x61, 0xc2, 0x99, 0x2f, 0x5e, 0xbc,

 0x65, 0xca, 0x89, 0x0f, 0x1e, 0x3c, 0x78, 0xf0,

 0xfd, 0xe7, 0xd3, 0xbb, 0x6b, 0xd6, 0xb1, 0x7f,

 0xfe, 0xe1, 0xdf, 0xa3, 0x5b, 0xb6, 0x71, 0xe2,

 0xd9, 0xaf, 0x43, 0x86, 0x11, 0x22, 0x44, 0x88,

 0x0d, 0x1a, 0x34, 0x68, 0xd0, 0xbd, 0x67, 0xce,

 0x81, 0x1f, 0x3e, 0x7c, 0xf8, 0xed, 0xc7, 0x93,

 0x3b, 0x76, 0xec, 0xc5, 0x97, 0x33, 0x66, 0xcc,

 0x85, 0x17, 0x2e, 0x5c, 0xb8, 0x6d, 0xda, 0xa9,

 0x4f, 0x9e, 0x21, 0x42, 0x84, 0x15, 0x2a, 0x54,

 0xa8, 0x4d, 0x9a, 0x29, 0x52, 0xa4, 0x55, 0xaa,

 0x49, 0x92, 0x39, 0x72, 0xe4, 0xd5, 0xb7, 0x73,

 0xe6, 0xd1, 0xbf, 0x63, 0xc6, 0x91, 0x3f, 0x7e,

 0xfc, 0xe5, 0xd7, 0xb3, 0x7b, 0xf6, 0xf1, 0xff,

 0xe3, 0xdb, 0xab, 0x4b, 0x96, 0x31, 0x62, 0xc4,

 0x95, 0x37, 0x6e, 0xdc, 0xa5, 0x57, 0xae, 0x41,

 0x82, 0x19, 0x32, 0x64, 0xc8, 0x8d, 0x07, 0x0e,

 0x1c, 0x38, 0x70, 0xe0, 0xdd, 0xa7, 0x53, 0xa6,

 0x51, 0xa2, 0x59, 0xb2, 0x79, 0xf2, 0xf9, 0xef,

 0xc3, 0x9b, 0x2b, 0x56, 0xac, 0x45, 0x8a, 0x09,

 0x12, 0x24, 0x48, 0x90, 0x3d, 0x7a, 0xf4, 0xf5,

 0xf7, 0xf3, 0xfb, 0xeb, 0xcb, 0x8b, 0x0b, 0x16,

 0x2c, 0x58, 0xb0, 0x7d, 0xfa, 0xe9, 0xcf, 0x83,

 0x1b, 0x36, 0x6c, 0xd8, 0xad, 0x47, 0x8e, 0x01

]

GF\_ADD : (GF28, GF28) -> GF28

GF\_ADD (x, y) = x ^ y

GF\_SUB : (GF28, GF28) -> GF28

GF\_SUB (x, y) = x ^ y

GF\_MUL : (GF28, GF28) -> GF28

GF\_MUL (x, y) = z

 where z = if x == 0 then 0

 else if y == 0 then 0

 else EXP @ ((toInteger (LOG @ x) + toInteger (LOG @ y)) % 255)

GF\_POW : (GF28, [8]) -> GF28

GF\_POW (n, k) = pows ! 0

 where pows = [1] # [ if bit then GF\_MUL (n, sq x)

 else sq x

 | x <- pows

 | bit <- k

 ]

 sq x = GF\_MUL (x, x)

GF\_DIV : (GF28, GF28) -> GF28

GF\_DIV (x, y) = div x

 where div i = if x == 0 then 0

 else EXP @ (((toInteger(LOG @ x)) - toInteger (LOG @ y)) % 255)

GF\_SUM : {n} (fin n) => [n]GF28 -> GF28

GF\_SUM ps = accum ! 0

 where accum = [zero] # [ GF\_ADD(p, a) | p <- ps | a <- accum ]

GF\_PROD : {n} (fin n) => [n]GF28 -> GF28

GF\_PROD ps = accum ! 0

 where accum = [1] # [ GF\_MUL(p, a) | p <- ps | a <- accum ]

GF\_DOT\_PROD : {n} (fin n) => ([n]GF28, [n]GF28) -> GF28

GF\_DOT\_PROD (xs, ys) = GF\_SUM [ GF\_MUL (x, y) | x <- xs

 | y <- ys

 ]

GF\_VEC\_MUL : {n, m} (fin n) => ([n]GF28, [m][n]GF28) -> [m]GF28

GF\_VEC\_MUL (v, ms) = [ GF\_DOT\_PROD(v, m) | m <- ms ]

GF\_MAT\_MUL : {n, m, k} (fin m) => ([n][m]GF28, [m][k]GF28) -> [n][k]GF28

GF\_MAT\_MUL (xss, yss) = [ GF\_VEC\_MUL(xs, yss') | xs <- xss ]

 where yss' = transpose yss

// Test test vectors for Polynomial 2 (x^^8 + x^^4 + x^^3 + x^^2 + 1)

/\*

 \* Test vector TV011D\_1

 \* secret = 74 65 73 74 00

 \* random = F3 C2 33 81 F5

 \*

 \* l = 5

 \* m = 2

 \* n = 2

 \*

 \* split1 = 87 A7 40 F5 F5

 \* split2 = 8F FC 15 6B F7

 \*/

TV011D\_TV1\_P : [2][2]GF28

TV011D\_TV1\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01) ]

]

TV011D\_TV1\_SR : [2][5]GF28

TV011D\_TV1\_SR = [

 [ 0x74, 0x65, 0x73, 0x74, 0x00 ],

 [ 0xF3, 0xC2, 0x33, 0x81, 0xF5 ]

]

TV011D\_TV1\_SPLITS : [2][5]GF28

TV011D\_TV1\_SPLITS = [

 [ 0x87, 0xA7, 0x40, 0xF5, 0xF5 ],

 [ 0x8F, 0xFC, 0x15, 0x6B, 0xF7 ]

]

/\* test split \*/

property TV011D\_TV1\_split\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV1\_P, TV011D\_TV1\_SR) == TV011D\_TV1\_SPLITS ] == ~zero

TV011D\_TV1\_1\_2\_R : [1][2]GF28

TV011D\_TV1\_1\_2\_R = [

 [ GF\_DIV(0x02, GF\_ADD(0x02, 0x01)), GF\_DIV(0x01, GF\_ADD(0x01, 0x02)) ]

]

TV011D\_TV1\_1\_2\_SPLITS : [2][5]GF28

TV011D\_TV1\_1\_2\_SPLITS = [

 [ 0x87, 0xA7, 0x40, 0xF5, 0xF5 ],

 [ 0x8F, 0xFC, 0x15, 0x6B, 0xF7 ]

]

TV011D\_TV1\_SECRET : [1][5]GF28

TV011D\_TV1\_SECRET = [

 [0x74, 0x65, 0x73, 0x74, 0x00 ]

]

/\* test recombine \*/

property TV011D\_TV1\_recombine\_1\_2\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV1\_1\_2\_R, TV011D\_TV1\_1\_2\_SPLITS) == TV011D\_TV1\_SECRET ] == ~zero

/\*

 \* Test vector TV011D\_2

 \* secret = 53 41 4D 54 43

 \* random = 20 76 08 93 0C

 \*

 \* l = 5

 \* m = 2

 \* n = 4

 \*

 \* split1 = 73 37 45 C7 4F

 \* split2 = 13 AD 5D 6F 5B

 \* split3 = 33 DB 55 FC 57

 \* split4 = D3 84 6D 22 73

 \*/

TV011D\_TV2\_P : [4][2]GF28

TV011D\_TV2\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01) ]

]

TV011D\_TV2\_SR : [2][5]GF28

TV011D\_TV2\_SR = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ],

 [ 0x20, 0x76, 0x08, 0x93, 0x0C ]

]

TV011D\_TV2\_SPLITS : [4][5]GF28

TV011D\_TV2\_SPLITS = [

 [ 0x73, 0x37, 0x45, 0xC7, 0x4F ],

 [ 0x13, 0xAD, 0x5D, 0x6F, 0x5B ],

 [ 0x33, 0xDB, 0x55, 0xFC, 0x57 ],

 [ 0xD3, 0x84, 0x6D, 0x22, 0x73 ]

]

/\* test split \*/

property TV011D\_TV2\_split\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV2\_P, TV011D\_TV2\_SR) == TV011D\_TV2\_SPLITS ] == ~zero

TV011D\_TV2\_1\_2\_R : [1][2]GF28

TV011D\_TV2\_1\_2\_R = [

 [ GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x01, GF\_ADD(0x01, 0x02)) ]

]

TV011D\_TV2\_1\_4\_R : [1][2]GF28

TV011D\_TV2\_1\_4\_R = [

 [ GF\_DIV(0x04, GF\_ADD(0x01, 0x04)), GF\_DIV(0x01, GF\_ADD(0x01, 0x04)) ]

]

TV011D\_TV2\_3\_4\_R : [1][2]GF28

TV011D\_TV2\_3\_4\_R = [

 [ GF\_DIV(0x04, GF\_ADD(0x03, 0x04)), GF\_DIV(0x03, GF\_ADD(0x03, 0x04)) ]

]

TV011D\_TV2\_1\_2\_SPLITS : [2][5]GF28

TV011D\_TV2\_1\_2\_SPLITS = [

 [ 0x73, 0x37, 0x45, 0xC7, 0x4F ],

 [ 0x13, 0xAD, 0x5D, 0x6F, 0x5B ]

]

TV011D\_TV2\_1\_4\_SPLITS : [2][5]GF28

TV011D\_TV2\_1\_4\_SPLITS = [

 [ 0x73, 0x37, 0x45, 0xC7, 0x4F ],

 [ 0xD3, 0x84, 0x6D, 0x22, 0x73 ]

]

TV011D\_TV2\_3\_4\_SPLITS : [2][5]GF28

TV011D\_TV2\_3\_4\_SPLITS = [

 [ 0x33, 0xDB, 0x55, 0xFC, 0x57 ],

 [ 0xD3, 0x84, 0x6D, 0x22, 0x73 ]

]

TV011D\_TV2\_SECRET : [1][5]GF28

TV011D\_TV2\_SECRET = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ]

]

/\* test recombines \*/

property TV011D\_TV2\_recombine\_1\_2\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV2\_1\_2\_R, TV011D\_TV2\_1\_2\_SPLITS) == TV011D\_TV2\_SECRET ] == ~zero

property TV011D\_TV2\_recombine\_1\_4\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV2\_1\_4\_R, TV011D\_TV2\_1\_4\_SPLITS) == TV011D\_TV2\_SECRET ] == ~zero

property TV011D\_TV2\_recombine\_3\_4\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV2\_3\_4\_R, TV011D\_TV2\_3\_4\_SPLITS) == TV011D\_TV2\_SECRET ] == ~zero

/\*

 \* Test vector TV011D\_3

 \* secret = 53 41 4D 54 43

 \* random = 8C 15 92 62 5C 4A AF 53 41 45

 \*

 \* l = 5

 \* m = 3

 \* n = 4

 \*

 \* split1 = CA B1 5B A8 47

 \* split2 = 02 ED C0 46 C8

 \* split3 = 9B 1D D6 BA CC

 \* split4 = 14 5D F4 8B 7E

 \*/

TV011D\_TV3\_P : [4][3]GF28

TV011D\_TV3\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01), GF\_POW(0x01, 0x02) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01), GF\_POW(0x02, 0x02) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01), GF\_POW(0x03, 0x02) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01), GF\_POW(0x04, 0x02) ]

]

TV011D\_TV3\_SR : [3][5]GF28

TV011D\_TV3\_SR = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ],

 [ 0x8C, 0x92, 0x5C, 0xAF, 0x41 ],

 [ 0x15, 0x62, 0x4A, 0x53, 0x45 ]

]

TV011D\_TV3\_SPLITS : [4][5]GF28

TV011D\_TV3\_SPLITS = [

 [ 0xCA, 0xB1, 0x5B, 0xA8, 0x47 ],

 [ 0x02, 0xED, 0xC0, 0x46, 0xC8 ],

 [ 0x9B, 0x1D, 0xD6, 0xBA, 0xCC ],

 [ 0x14, 0x5D, 0xF4, 0x8B, 0x7E ]

]

/\* test split \*/

property TV011D\_TV3\_split\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV3\_P, TV011D\_TV3\_SR) == TV011D\_TV3\_SPLITS ] == ~zero

TV011D\_TV3\_1\_2\_3\_R : [1][3]GF28

TV011D\_TV3\_1\_2\_3\_R = [

 [ GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x01, 0x03))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x02, 0x03))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x03)), GF\_DIV(0x02, GF\_ADD(0x02, 0x03))]

 ]

]

TV011D\_TV3\_1\_2\_4\_R : [1][3]GF28

TV011D\_TV3\_1\_2\_4\_R = [

 [ GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x04, GF\_ADD(0x01, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x02)), GF\_DIV(0x04, GF\_ADD(0x02, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x04)), GF\_DIV(0x02, GF\_ADD(0x02, 0x04))]

 ]

]

TV011D\_TV3\_1\_3\_4\_R : [1][3]GF28

TV011D\_TV3\_1\_3\_4\_R = [

 [ GF\_PROD[GF\_DIV(0x03, GF\_ADD(0x01, 0x03)), GF\_DIV(0x04, GF\_ADD(0x01, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x03)), GF\_DIV(0x04, GF\_ADD(0x03, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x04)), GF\_DIV(0x03, GF\_ADD(0x03, 0x04))]

 ]

]

TV011D\_TV3\_1\_2\_3\_SPLITS : [3][5]GF28

TV011D\_TV3\_1\_2\_3\_SPLITS = [

 [ 0xCA, 0xB1, 0x5B, 0xA8, 0x47 ],

 [ 0x02, 0xED, 0xC0, 0x46, 0xC8 ],

 [ 0x9B, 0x1D, 0xD6, 0xBA, 0xCC ]

]

TV011D\_TV3\_1\_2\_4\_SPLITS : [3][5]GF28

TV011D\_TV3\_1\_2\_4\_SPLITS = [

 [ 0xCA, 0xB1, 0x5B, 0xA8, 0x47 ],

 [ 0x02, 0xED, 0xC0, 0x46, 0xC8 ],

 [ 0x14, 0x5D, 0xF4, 0x8B, 0x7E ]

]

TV011D\_TV3\_1\_3\_4\_SPLITS : [3][5]GF28

TV011D\_TV3\_1\_3\_4\_SPLITS = [

 [ 0xCA, 0xB1, 0x5B, 0xA8, 0x47 ],

 [ 0x9B, 0x1D, 0xD6, 0xBA, 0xCC ],

 [ 0x14, 0x5D, 0xF4, 0x8B, 0x7E ]

]

TV011D\_TV3\_SECRET : [1][5]GF28

TV011D\_TV3\_SECRET = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ]

]

/\* test recombines \*/

property TV011D\_TV3\_recombine\_1\_2\_3\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV3\_1\_2\_3\_R, TV011D\_TV3\_1\_2\_3\_SPLITS) == TV011D\_TV3\_SECRET ] == ~zero

property TV011D\_TV3\_recombine\_1\_2\_4\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV3\_1\_2\_4\_R, TV011D\_TV3\_1\_2\_4\_SPLITS) == TV011D\_TV3\_SECRET ] == ~zero

property TV011D\_TV3\_recombine\_1\_3\_4\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV3\_1\_3\_4\_R, TV011D\_TV3\_1\_3\_4\_SPLITS) == TV011D\_TV3\_SECRET ] == ~zero

/\*

 \* Test vector TV011D\_4

 \* secret = 53 41 4D 54 43

 \* random = 72 B0 88 3C 96 B9 CB B9 CB B2 82 66 F3 79 FA

 \*

 \* l = 5

 \* m = 4

 \* n = 4

 \*

 \* split1 = 19 52 F4 02 33

 \* split2 = 79 FA 0E 08 C2

 \* split3 = 24 58 37 17 94

 \* split4 = F4 45 A9 D6 07

 \*/

TV011D\_TV4\_P : [4][4]GF28

TV011D\_TV4\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01), GF\_POW(0x01, 0x02), GF\_POW(0x01, 0x03) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01), GF\_POW(0x02, 0x02), GF\_POW(0x02, 0x03) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01), GF\_POW(0x03, 0x02), GF\_POW(0x03, 0x03) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01), GF\_POW(0x04, 0x02), GF\_POW(0x04, 0x03) ]

]

TV011D\_TV4\_SR : [4][5]GF28

TV011D\_TV4\_SR = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ],

 [ 0x72, 0x3C, 0xCB, 0xB2, 0xF3 ],

 [ 0xB0, 0x96, 0xB9, 0x82, 0x79 ],

 [ 0x88, 0xB9, 0xCB, 0x66, 0xFA ]

]

TV011D\_TV4\_SPLITS : [4][5]GF28

TV011D\_TV4\_SPLITS = [

 [ 0x19, 0x52, 0xF4, 0x02, 0x33 ],

 [ 0x79, 0xFA, 0x0E, 0x08, 0xC2 ],

 [ 0x24, 0x58, 0x37, 0x17, 0x94 ],

 [ 0xF4, 0x45, 0xA9, 0xD6, 0x07 ]

]

/\* test split \*/

property TV011D\_TV4\_split\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV4\_P, TV011D\_TV4\_SR) == TV011D\_TV4\_SPLITS ] == ~zero

TV011D\_TV4\_1\_2\_3\_4\_R : [1][4]GF28

TV011D\_TV4\_1\_2\_3\_4\_R = [

 [ GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x01, 0x03)), GF\_DIV(0x04, GF\_ADD(0x01, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x02, 0x03)), GF\_DIV(0x04, GF\_ADD(0x02, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x03)), GF\_DIV(0x02, GF\_ADD(0x02, 0x03)), GF\_DIV(0x04, GF\_ADD(0x03, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x04)), GF\_DIV(0x02, GF\_ADD(0x02, 0x04)), GF\_DIV(0x03, GF\_ADD(0x03, 0x04))]

 ]

]

TV011D\_TV4\_1\_2\_3\_4\_SPLITS : [4][5]GF28

TV011D\_TV4\_1\_2\_3\_4\_SPLITS = [

 [ 0x19, 0x52, 0xF4, 0x02, 0x33 ],

 [ 0x79, 0xFA, 0x0E, 0x08, 0xC2 ],

 [ 0x24, 0x58, 0x37, 0x17, 0x94 ],

 [ 0xF4, 0x45, 0xA9, 0xD6, 0x07 ]

]

TV011D\_TV4\_SECRET : [1][5]GF28

TV011D\_TV4\_SECRET = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ]

]

/\* test recombines \*/

property TV011D\_TV4\_recombine\_1\_2\_3\_4\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV4\_1\_2\_3\_4\_R, TV011D\_TV4\_1\_2\_3\_4\_SPLITS) == TV011D\_TV4\_SECRET ] == ~zero

/\*

 \* Test vector TV011D\_5

 \* secret = 54 65 73 74 20 44 61 74 61

 \* random = AF FD 2B 0B FA 34 33 63 9C

 \*

 \* l = 9

 \* m = 2

 \* n = 9

 \*

 \* split1 = FB 98 58 7F DA 70 52 17 FD

 \* split2 = 17 82 25 62 C9 2C 07 B2 44

 \* split3 = B8 7F 0E 69 33 18 34 D1 D8

 \* split4 = D2 B6 DF 58 EF 94 AD E5 2B

 \* split5 = 7D 4B F4 53 15 A0 9E 86 B7

 \* split6 = 91 51 89 4E 06 FC CB 23 0E

 \* split7 = 3E AC A2 45 FC C8 F8 40 92

 \* split8 = 45 DE 36 2C A3 F9 E4 4B F5

 \* split9 = EA 23 1D 27 59 CD D7 28 69

 \*/

TV011D\_TV5\_P : [9][2]GF28

TV011D\_TV5\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01) ],

 [ GF\_POW(0x05, 0x00), GF\_POW(0x05, 0x01) ],

 [ GF\_POW(0x06, 0x00), GF\_POW(0x06, 0x01) ],

 [ GF\_POW(0x07, 0x00), GF\_POW(0x07, 0x01) ],

 [ GF\_POW(0x08, 0x00), GF\_POW(0x08, 0x01) ],

 [ GF\_POW(0x09, 0x00), GF\_POW(0x09, 0x01) ]

]

TV011D\_TV5\_SR : [2][9]GF28

TV011D\_TV5\_SR = [

 [ 0x54, 0x65, 0x73, 0x74, 0x20, 0x44, 0x61, 0x74, 0x61 ],

 [ 0xAF, 0xFD, 0x2B, 0x0B, 0xFA, 0x34, 0x33, 0x63, 0x9C ]

]

TV011D\_TV5\_SPLITS : [9][9]GF28

TV011D\_TV5\_SPLITS = [

 [ 0xFB, 0x98, 0x58, 0x7F, 0xDA, 0x70, 0x52, 0x17, 0xFD ],

 [ 0x17, 0x82, 0x25, 0x62, 0xC9, 0x2C, 0x07, 0xB2, 0x44 ],

 [ 0xB8, 0x7F, 0x0E, 0x69, 0x33, 0x18, 0x34, 0xD1, 0xD8 ],

 [ 0xD2, 0xB6, 0xDF, 0x58, 0xEF, 0x94, 0xAD, 0xE5, 0x2B ],

 [ 0x7D, 0x4B, 0xF4, 0x53, 0x15, 0xA0, 0x9E, 0x86, 0xB7 ],

 [ 0x91, 0x51, 0x89, 0x4E, 0x06, 0xFC, 0xCB, 0x23, 0x0E ],

 [ 0x3E, 0xAC, 0xA2, 0x45, 0xFC, 0xC8, 0xF8, 0x40, 0x92 ],

 [ 0x45, 0xDE, 0x36, 0x2C, 0xA3, 0xF9, 0xE4, 0x4B, 0xF5 ],

 [ 0xEA, 0x23, 0x1D, 0x27, 0x59, 0xCD, 0xD7, 0x28, 0x69 ]

]

/\* test split \*/

property TV011D\_TV5\_split\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV5\_P, TV011D\_TV5\_SR) == TV011D\_TV5\_SPLITS ] == ~zero

TV011D\_TV5\_1\_2\_R : [1][2]GF28

TV011D\_TV5\_1\_2\_R = [

 [ GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x01, GF\_ADD(0x01, 0x02)) ]

]

TV011D\_TV5\_8\_9\_R : [1][2]GF28

TV011D\_TV5\_8\_9\_R = [

 [ GF\_DIV(0x09, GF\_ADD(0x08, 0x09)), GF\_DIV(0x08, GF\_ADD(0x08, 0x09)) ]

]

TV011D\_TV5\_1\_2\_SPLITS : [2][9]GF28

TV011D\_TV5\_1\_2\_SPLITS = [

 [ 0xFB, 0x98, 0x58, 0x7F, 0xDA, 0x70, 0x52, 0x17, 0xFD ],

 [ 0x17, 0x82, 0x25, 0x62, 0xC9, 0x2C, 0x07, 0xB2, 0x44 ]

]

TV011D\_TV5\_8\_9\_SPLITS : [2][9]GF28

TV011D\_TV5\_8\_9\_SPLITS = [

 [ 0x45, 0xDE, 0x36, 0x2C, 0xA3, 0xF9, 0xE4, 0x4B, 0xF5 ],

 [ 0xEA, 0x23, 0x1D, 0x27, 0x59, 0xCD, 0xD7, 0x28, 0x69 ]

]

TV011D\_TV5\_SECRET : [1][9]GF28

TV011D\_TV5\_SECRET = [

 [ 0x54, 0x65, 0x73, 0x74, 0x20, 0x44, 0x61, 0x74, 0x61 ]

]

/\* test recombines \*/

property TV011D\_TV5\_recombine\_1\_2\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV5\_1\_2\_R, TV011D\_TV5\_1\_2\_SPLITS) == TV011D\_TV5\_SECRET ] == ~zero

property TV011D\_TV5\_recombine\_8\_9\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV5\_8\_9\_R, TV011D\_TV5\_8\_9\_SPLITS) == TV011D\_TV5\_SECRET ] == ~zero

/\*

 \* Test vector TV011D\_6

 \* secret = 01 02 03 04 05 06 07 08 09 0A 0B 0C 0D 0E 0F

 \* random = 02 4A 89 AC 96 8C 98 65 77 FE B0 24 11 6B 94 F6 54 DD DE 20 9C 3C C3 E4 48 88 4D 31 F8 C8

 \*

 \* l = 15

 \* m = 3

 \* n = 5

 \*

 \* split1 = 49 27 19 F9 8C 92 7D 6A 80 F4 AB 2B CD 72 3F

 \* split2 = 30 87 38 A0 34 EB 94 C2 F2 2B DE 20 87 50 E5

 \* split3 = 78 A2 22 5D BD 7F EE A0 7B D5 7E 07 47 2C D5

 \* split4 = DD 0E 49 40 9F 86 BD B9 15 6F A6 C1 58 10 D4

 \* split5 = 95 2B 53 BD 16 12 C7 DB 9C 91 06 E6 98 6C E4

 \*/

TV011D\_TV6\_P : [5][3]GF28

TV011D\_TV6\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01), GF\_POW(0x01, 0x02) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01), GF\_POW(0x02, 0x02) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01), GF\_POW(0x03, 0x02) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01), GF\_POW(0x04, 0x02) ],

 [ GF\_POW(0x05, 0x00), GF\_POW(0x05, 0x01), GF\_POW(0x05, 0x02) ]

]

TV011D\_TV6\_SR : [3][15]GF28

TV011D\_TV6\_SR = [

 [ 0x01, 0x02, 0x03, 0x04, 0x05, 0x06, 0x07, 0x08, 0x09, 0x0A, 0x0B, 0x0C, 0x0D, 0x0E, 0x0F ],

 [ 0x02, 0x89, 0x96, 0x98, 0x77, 0xB0, 0x11, 0x94, 0x54, 0xDE, 0x9C, 0xC3, 0x48, 0x4D, 0xF8 ],

 [ 0x4A, 0xAC, 0x8C, 0x65, 0xFE, 0x24, 0x6B, 0xF6, 0xDD, 0x20, 0x3C, 0xE4, 0x88, 0x31, 0xC8 ]

]

TV011D\_TV6\_SPLITS : [5][15]GF28

TV011D\_TV6\_SPLITS = [

 [ 0x49, 0x27, 0x19, 0xF9, 0x8C, 0x92, 0x7D, 0x6A, 0x80, 0xF4, 0xAB, 0x2B, 0xCD, 0x72, 0x3F ],

 [ 0x30, 0x87, 0x38, 0xA0, 0x34, 0xEB, 0x94, 0xC2, 0xF2, 0x2B, 0xDE, 0x20, 0x87, 0x50, 0xE5 ],

 [ 0x78, 0xA2, 0x22, 0x5D, 0xBD, 0x7F, 0xEE, 0xA0, 0x7B, 0xD5, 0x7E, 0x07, 0x47, 0x2C, 0xD5 ],

 [ 0xDD, 0x0E, 0x49, 0x40, 0x9F, 0x86, 0xBD, 0xB9, 0x15, 0x6F, 0xA6, 0xC1, 0x58, 0x10, 0xD4 ],

 [ 0x95, 0x2B, 0x53, 0xBD, 0x16, 0x12, 0xC7, 0xDB, 0x9C, 0x91, 0x06, 0xE6, 0x98, 0x6C, 0xE4 ]

]

/\* test split \*/

property TV011D\_TV6\_split\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV6\_P, TV011D\_TV6\_SR) == TV011D\_TV6\_SPLITS ] == ~zero

TV011D\_TV6\_1\_2\_3\_R : [1][3]GF28

TV011D\_TV6\_1\_2\_3\_R = [

 [ GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x01, 0x03))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x02, 0x03))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x03)), GF\_DIV(0x02, GF\_ADD(0x02, 0x03))]

 ]

]

TV011D\_TV6\_2\_3\_4\_R : [1][3]GF28

TV011D\_TV6\_2\_3\_4\_R = [

 [ GF\_PROD[GF\_DIV(0x03, GF\_ADD(0x02, 0x03)), GF\_DIV(0x04, GF\_ADD(0x02, 0x04))],

 GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x02, 0x03)), GF\_DIV(0x04, GF\_ADD(0x03, 0x04))],

 GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x02, 0x04)), GF\_DIV(0x03, GF\_ADD(0x03, 0x04))]

 ]

]

TV011D\_TV6\_1\_2\_3\_SPLITS : [3][15]GF28

TV011D\_TV6\_1\_2\_3\_SPLITS = [

 [ 0x49, 0x27, 0x19, 0xF9, 0x8C, 0x92, 0x7D, 0x6A, 0x80, 0xF4, 0xAB, 0x2B, 0xCD, 0x72, 0x3F ],

 [ 0x30, 0x87, 0x38, 0xA0, 0x34, 0xEB, 0x94, 0xC2, 0xF2, 0x2B, 0xDE, 0x20, 0x87, 0x50, 0xE5 ],

 [ 0x78, 0xA2, 0x22, 0x5D, 0xBD, 0x7F, 0xEE, 0xA0, 0x7B, 0xD5, 0x7E, 0x07, 0x47, 0x2C, 0xD5 ]

]

TV011D\_TV6\_2\_3\_4\_SPLITS : [3][15]GF28

TV011D\_TV6\_2\_3\_4\_SPLITS = [

 [ 0x30, 0x87, 0x38, 0xA0, 0x34, 0xEB, 0x94, 0xC2, 0xF2, 0x2B, 0xDE, 0x20, 0x87, 0x50, 0xE5 ],

 [ 0x78, 0xA2, 0x22, 0x5D, 0xBD, 0x7F, 0xEE, 0xA0, 0x7B, 0xD5, 0x7E, 0x07, 0x47, 0x2C, 0xD5 ],

 [ 0xDD, 0x0E, 0x49, 0x40, 0x9F, 0x86, 0xBD, 0xB9, 0x15, 0x6F, 0xA6, 0xC1, 0x58, 0x10, 0xD4 ]

]

TV011D\_TV6\_SECRET : [1][15]GF28

TV011D\_TV6\_SECRET = [

 [ 0x01, 0x02, 0x03, 0x04, 0x05, 0x06, 0x07, 0x08, 0x09, 0x0A, 0x0B, 0x0C, 0x0D, 0x0E, 0x0F ]

]

/\* test recombines \*/

property TV011D\_TV6\_recombine\_1\_2\_3\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV6\_1\_2\_3\_R, TV011D\_TV6\_1\_2\_3\_SPLITS) == TV011D\_TV6\_SECRET ] == ~zero

property TV011D\_TV6\_recombine\_2\_3\_4\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV6\_2\_3\_4\_R, TV011D\_TV6\_2\_3\_4\_SPLITS) == TV011D\_TV6\_SECRET ] == ~zero

* + 1. “Native” GF(256)-based implementation

module secret\_share\_011D\_gf28 where

type GF28 = [8]

irreducible = <| x^^8 + x^^4 + x^^3 + x^^2 + 1 |>

GF\_ADD : (GF28, GF28) -> GF28

GF\_ADD (x, y) = x ^ y

GF\_SUB : (GF28, GF28) -> GF28

GF\_SUB (x, y) = x ^ y

GF\_MUL : (GF28, GF28) -> GF28

GF\_MUL (x, y) = pmod(pmult x y) irreducible

GF\_POW : (GF28, [8]) -> GF28

GF\_POW (n, k) = pow k

 where sq x = GF\_MUL (x, x)

 odd x = x ! 0

 pow i = if i == 0 then 1

 else if odd i

 then GF\_MUL(n, sq (pow (i >> 1)))

 else sq (pow (i >> 1))

GF\_INV : GF28 -> GF28

GF\_INV x = GF\_POW (x, 254)

GF\_DIV : (GF28, GF28) -> GF28

GF\_DIV (x, y) = div x

 where inv yi = GF\_INV (yi)

 div i = if x == 0 then 0

 else GF\_MUL(x, inv y)

GF\_SUM : {n} (fin n) => [n]GF28 -> GF28

GF\_SUM ps = accum ! 0

 where accum = [zero] # [ GF\_ADD(p, a) | p <- ps | s <- accum ]

GF\_PROD : {n} (fin n) => [n]GF28 -> GF28

GF\_PROD ps = accum ! 0

 where accum = [1] # [ GF\_MUL(p, a) | p <- ps | a <- accum ]

GF\_DOT\_PROD : {n} (fin n) => ([n]GF28, [n]GF28) -> GF28

GF\_DOT\_PROD (xs, ys) = GF\_SUM [ GF\_MUL (x, y) | x <- xs

 | y <- ys

 ]

GF\_VEC\_MUL : {n, m} (fin n) => ([n]GF28, [m][n]GF28) -> [m]GF28

GF\_VEC\_MUL (v, ms) = [ GF\_DOT\_PROD(v, m) | m <- ms ]

GF\_MAT\_MUL : {n, m, k} (fin m) => ([n][m]GF28, [m][k]GF28) -> [n][k]GF28

GF\_MAT\_MUL (xss, yss) = [ GF\_VEC\_MUL(xs, yss') | xs <- xss ]

 where yss' = transpose yss

// Test test vectors for Polynomial 2 (x^^8 + x^^4 + x^^3 + x^^2 + 1)

/\*

 \* Test vector TV011D\_1

 \* secret = 74 65 73 74 00

 \* random = F3 C2 33 81 F5

 \*

 \* l = 5

 \* m = 2

 \* n = 2

 \*

 \* split1 = 87 A7 40 F5 F5

 \* split2 = 8F FC 15 6B F7

 \*/

TV011D\_TV1\_P : [2][2]GF28

TV011D\_TV1\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01) ]

]

TV011D\_TV1\_SR : [2][5]GF28

TV011D\_TV1\_SR = [

 [ 0x74, 0x65, 0x73, 0x74, 0x00 ],

 [ 0xF3, 0xC2, 0x33, 0x81, 0xF5 ]

]

TV011D\_TV1\_SPLITS : [2][5]GF28

TV011D\_TV1\_SPLITS = [

 [ 0x87, 0xA7, 0x40, 0xF5, 0xF5 ],

 [ 0x8F, 0xFC, 0x15, 0x6B, 0xF7 ]

]

/\* test split \*/

property TV011D\_TV1\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV1\_P, TV011D\_TV1\_SR) == TV011D\_TV1\_SPLITS ] == ~zero

TV011D\_TV1\_1\_2\_R : [1][2]GF28

TV011D\_TV1\_1\_2\_R = [

 [ GF\_DIV(0x02, GF\_ADD(0x02, 0x01)), GF\_DIV(0x01, GF\_ADD(0x01, 0x02)) ]

]

TV011D\_TV1\_1\_2\_SPLITS : [2][5]GF28

TV011D\_TV1\_1\_2\_SPLITS = [

 [ 0x87, 0xA7, 0x40, 0xF5, 0xF5 ],

 [ 0x8F, 0xFC, 0x15, 0x6B, 0xF7 ]

]

TV011D\_TV1\_SECRET : [1][5]GF28

TV011D\_TV1\_SECRET = [

 [0x74, 0x65, 0x73, 0x74, 0x00 ]

]

/\* test recombine \*/

property TV011D\_TV1\_recombine\_1\_2\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV1\_1\_2\_R, TV011D\_TV1\_1\_2\_SPLITS) == TV011D\_TV1\_SECRET ] == ~zero

/\*

 \* Test vector TV011D\_2

 \* secret = 53 41 4D 54 43

 \* random = 20 76 08 93 0C

 \*

 \* l = 5

 \* m = 2

 \* n = 4

 \*

 \* split1 = 73 37 45 C7 4F

 \* split2 = 13 AD 5D 6F 5B

 \* split3 = 33 DB 55 FC 57

 \* split4 = D3 84 6D 22 73

 \*/

TV011D\_TV2\_P : [4][2]GF28

TV011D\_TV2\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01) ]

]

TV011D\_TV2\_SR : [2][5]GF28

TV011D\_TV2\_SR = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ],

 [ 0x20, 0x76, 0x08, 0x93, 0x0C ]

]

TV011D\_TV2\_SPLITS : [4][5]GF28

TV011D\_TV2\_SPLITS = [

 [ 0x73, 0x37, 0x45, 0xC7, 0x4F ],

 [ 0x13, 0xAD, 0x5D, 0x6F, 0x5B ],

 [ 0x33, 0xDB, 0x55, 0xFC, 0x57 ],

 [ 0xD3, 0x84, 0x6D, 0x22, 0x73 ]

]

/\* test split \*/

property TV011D\_TV2\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV2\_P, TV011D\_TV2\_SR) == TV011D\_TV2\_SPLITS ] == ~zero

TV011D\_TV2\_1\_2\_R : [1][2]GF28

TV011D\_TV2\_1\_2\_R = [

 [ GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x01, GF\_ADD(0x01, 0x02)) ]

]

TV011D\_TV2\_1\_4\_R : [1][2]GF28

TV011D\_TV2\_1\_4\_R = [

 [ GF\_DIV(0x04, GF\_ADD(0x01, 0x04)), GF\_DIV(0x01, GF\_ADD(0x01, 0x04)) ]

]

TV011D\_TV2\_3\_4\_R : [1][2]GF28

TV011D\_TV2\_3\_4\_R = [

 [ GF\_DIV(0x04, GF\_ADD(0x03, 0x04)), GF\_DIV(0x03, GF\_ADD(0x03, 0x04)) ]

]

TV011D\_TV2\_1\_2\_SPLITS : [2][5]GF28

TV011D\_TV2\_1\_2\_SPLITS = [

 [ 0x73, 0x37, 0x45, 0xC7, 0x4F ],

 [ 0x13, 0xAD, 0x5D, 0x6F, 0x5B ]

]

TV011D\_TV2\_1\_4\_SPLITS : [2][5]GF28

TV011D\_TV2\_1\_4\_SPLITS = [

 [ 0x73, 0x37, 0x45, 0xC7, 0x4F ],

 [ 0xD3, 0x84, 0x6D, 0x22, 0x73 ]

]

TV011D\_TV2\_3\_4\_SPLITS : [2][5]GF28

TV011D\_TV2\_3\_4\_SPLITS = [

 [ 0x33, 0xDB, 0x55, 0xFC, 0x57 ],

 [ 0xD3, 0x84, 0x6D, 0x22, 0x73 ]

]

TV011D\_TV2\_SECRET : [1][5]GF28

TV011D\_TV2\_SECRET = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ]

]

/\* test recombines \*/

property TV011D\_TV2\_recombine\_1\_2\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV2\_1\_2\_R, TV011D\_TV2\_1\_2\_SPLITS) == TV011D\_TV2\_SECRET ] == ~zero

property TV011D\_TV2\_recombine\_1\_4\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV2\_1\_4\_R, TV011D\_TV2\_1\_4\_SPLITS) == TV011D\_TV2\_SECRET ] == ~zero

property TV011D\_TV2\_recombine\_3\_4\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV2\_3\_4\_R, TV011D\_TV2\_3\_4\_SPLITS) == TV011D\_TV2\_SECRET ] == ~zero

/\*

 \* Test vector TV011D\_3

 \* secret = 53 41 4D 54 43

 \* random = 8C 15 92 62 5C 4A AF 53 41 45

 \*

 \* l = 5

 \* m = 3

 \* n = 4

 \*

 \* split1 = CA B1 5B A8 47

 \* split2 = 02 ED C0 46 C8

 \* split3 = 9B 1D D6 BA CC

 \* split4 = 14 5D F4 8B 7E

 \*/

TV011D\_TV3\_P : [4][3]GF28

TV011D\_TV3\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01), GF\_POW(0x01, 0x02) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01), GF\_POW(0x02, 0x02) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01), GF\_POW(0x03, 0x02) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01), GF\_POW(0x04, 0x02) ]

]

TV011D\_TV3\_SR : [3][5]GF28

TV011D\_TV3\_SR = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ],

 [ 0x8C, 0x92, 0x5C, 0xAF, 0x41 ],

 [ 0x15, 0x62, 0x4A, 0x53, 0x45 ]

]

TV011D\_TV3\_SPLITS : [4][5]GF28

TV011D\_TV3\_SPLITS = [

 [ 0xCA, 0xB1, 0x5B, 0xA8, 0x47 ],

 [ 0x02, 0xED, 0xC0, 0x46, 0xC8 ],

 [ 0x9B, 0x1D, 0xD6, 0xBA, 0xCC ],

 [ 0x14, 0x5D, 0xF4, 0x8B, 0x7E ]

]

/\* test split \*/

property TV011D\_TV3\_split\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV3\_P, TV011D\_TV3\_SR) == TV011D\_TV3\_SPLITS ] == ~zero

TV011D\_TV3\_1\_2\_3\_R : [1][3]GF28

TV011D\_TV3\_1\_2\_3\_R = [

 [ GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x01, 0x03))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x02, 0x03))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x03)), GF\_DIV(0x02, GF\_ADD(0x02, 0x03))]

 ]

]

TV011D\_TV3\_1\_2\_4\_R : [1][3]GF28

TV011D\_TV3\_1\_2\_4\_R = [

 [ GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x04, GF\_ADD(0x01, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x02)), GF\_DIV(0x04, GF\_ADD(0x02, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x04)), GF\_DIV(0x02, GF\_ADD(0x02, 0x04))]

 ]

]

TV011D\_TV3\_1\_3\_4\_R : [1][3]GF28

TV011D\_TV3\_1\_3\_4\_R = [

 [ GF\_PROD[GF\_DIV(0x03, GF\_ADD(0x01, 0x03)), GF\_DIV(0x04, GF\_ADD(0x01, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x03)), GF\_DIV(0x04, GF\_ADD(0x03, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x04)), GF\_DIV(0x03, GF\_ADD(0x03, 0x04))]

 ]

]

TV011D\_TV3\_1\_2\_3\_SPLITS : [3][5]GF28

TV011D\_TV3\_1\_2\_3\_SPLITS = [

 [ 0xCA, 0xB1, 0x5B, 0xA8, 0x47 ],

 [ 0x02, 0xED, 0xC0, 0x46, 0xC8 ],

 [ 0x9B, 0x1D, 0xD6, 0xBA, 0xCC ]

]

TV011D\_TV3\_1\_2\_4\_SPLITS : [3][5]GF28

TV011D\_TV3\_1\_2\_4\_SPLITS = [

 [ 0xCA, 0xB1, 0x5B, 0xA8, 0x47 ],

 [ 0x02, 0xED, 0xC0, 0x46, 0xC8 ],

 [ 0x14, 0x5D, 0xF4, 0x8B, 0x7E ]

]

TV011D\_TV3\_1\_3\_4\_SPLITS : [3][5]GF28

TV011D\_TV3\_1\_3\_4\_SPLITS = [

 [ 0xCA, 0xB1, 0x5B, 0xA8, 0x47 ],

 [ 0x9B, 0x1D, 0xD6, 0xBA, 0xCC ],

 [ 0x14, 0x5D, 0xF4, 0x8B, 0x7E ]

]

TV011D\_TV3\_SECRET : [1][5]GF28

TV011D\_TV3\_SECRET = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ]

]

/\* test recombines \*/

property TV011D\_TV3\_recombine\_1\_2\_3\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV3\_1\_2\_3\_R, TV011D\_TV3\_1\_2\_3\_SPLITS) == TV011D\_TV3\_SECRET ] == ~zero

property TV011D\_TV3\_recombine\_1\_2\_4\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV3\_1\_2\_4\_R, TV011D\_TV3\_1\_2\_4\_SPLITS) == TV011D\_TV3\_SECRET ] == ~zero

property TV011D\_TV3\_recombine\_1\_3\_4\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV3\_1\_3\_4\_R, TV011D\_TV3\_1\_3\_4\_SPLITS) == TV011D\_TV3\_SECRET ] == ~zero

/\*

 \* Test vector TV011D\_4

 \* secret = 53 41 4D 54 43

 \* random = 72 B0 88 3C 96 B9 CB B9 CB B2 82 66 F3 79 FA

 \*

 \* l = 5

 \* m = 4

 \* n = 4

 \*

 \* split1 = 19 52 F4 02 33

 \* split2 = 79 FA 0E 08 C2

 \* split3 = 24 58 37 17 94

 \* split4 = F4 45 A9 D6 07

 \*/

TV011D\_TV4\_P : [4][4]GF28

TV011D\_TV4\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01), GF\_POW(0x01, 0x02), GF\_POW(0x01, 0x03) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01), GF\_POW(0x02, 0x02), GF\_POW(0x02, 0x03) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01), GF\_POW(0x03, 0x02), GF\_POW(0x03, 0x03) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01), GF\_POW(0x04, 0x02), GF\_POW(0x04, 0x03) ]

]

TV011D\_TV4\_SR : [4][5]GF28

TV011D\_TV4\_SR = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ],

 [ 0x72, 0x3C, 0xCB, 0xB2, 0xF3 ],

 [ 0xB0, 0x96, 0xB9, 0x82, 0x79 ],

 [ 0x88, 0xB9, 0xCB, 0x66, 0xFA ]

]

TV011D\_TV4\_SPLITS : [4][5]GF28

TV011D\_TV4\_SPLITS = [

 [ 0x19, 0x52, 0xF4, 0x02, 0x33 ],

 [ 0x79, 0xFA, 0x0E, 0x08, 0xC2 ],

 [ 0x24, 0x58, 0x37, 0x17, 0x94 ],

 [ 0xF4, 0x45, 0xA9, 0xD6, 0x07 ]

]

/\* test split \*/

property TV011D\_TV4\_split\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV4\_P, TV011D\_TV4\_SR) == TV011D\_TV4\_SPLITS ] == ~zero

TV011D\_TV4\_1\_2\_3\_4\_R : [1][4]GF28

TV011D\_TV4\_1\_2\_3\_4\_R = [

 [ GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x01, 0x03)), GF\_DIV(0x04, GF\_ADD(0x01, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x02, 0x03)), GF\_DIV(0x04, GF\_ADD(0x02, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x03)), GF\_DIV(0x02, GF\_ADD(0x02, 0x03)), GF\_DIV(0x04, GF\_ADD(0x03, 0x04))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x04)), GF\_DIV(0x02, GF\_ADD(0x02, 0x04)), GF\_DIV(0x03, GF\_ADD(0x03, 0x04))]

 ]

]

TV011D\_TV4\_1\_2\_3\_4\_SPLITS : [4][5]GF28

TV011D\_TV4\_1\_2\_3\_4\_SPLITS = [

 [ 0x19, 0x52, 0xF4, 0x02, 0x33 ],

 [ 0x79, 0xFA, 0x0E, 0x08, 0xC2 ],

 [ 0x24, 0x58, 0x37, 0x17, 0x94 ],

 [ 0xF4, 0x45, 0xA9, 0xD6, 0x07 ]

]

TV011D\_TV4\_SECRET : [1][5]GF28

TV011D\_TV4\_SECRET = [

 [ 0x53, 0x41, 0x4D, 0x54, 0x43 ]

]

/\* test recombines \*/

property TV011D\_TV4\_recombine\_1\_2\_3\_4\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV4\_1\_2\_3\_4\_R, TV011D\_TV4\_1\_2\_3\_4\_SPLITS) == TV011D\_TV4\_SECRET ] == ~zero

/\*

 \* Test vector TV011D\_5

 \* secret = 54 65 73 74 20 44 61 74 61

 \* random = AF FD 2B 0B FA 34 33 63 9C

 \*

 \* l = 9

 \* m = 2

 \* n = 9

 \*

 \* split1 = FB 98 58 7F DA 70 52 17 FD

 \* split2 = 17 82 25 62 C9 2C 07 B2 44

 \* split3 = B8 7F 0E 69 33 18 34 D1 D8

 \* split4 = D2 B6 DF 58 EF 94 AD E5 2B

 \* split5 = 7D 4B F4 53 15 A0 9E 86 B7

 \* split6 = 91 51 89 4E 06 FC CB 23 0E

 \* split7 = 3E AC A2 45 FC C8 F8 40 92

 \* split8 = 45 DE 36 2C A3 F9 E4 4B F5

 \* split9 = EA 23 1D 27 59 CD D7 28 69

 \*/

TV011D\_TV5\_P : [9][2]GF28

TV011D\_TV5\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01) ],

 [ GF\_POW(0x05, 0x00), GF\_POW(0x05, 0x01) ],

 [ GF\_POW(0x06, 0x00), GF\_POW(0x06, 0x01) ],

 [ GF\_POW(0x07, 0x00), GF\_POW(0x07, 0x01) ],

 [ GF\_POW(0x08, 0x00), GF\_POW(0x08, 0x01) ],

 [ GF\_POW(0x09, 0x00), GF\_POW(0x09, 0x01) ]

]

TV011D\_TV5\_SR : [2][9]GF28

TV011D\_TV5\_SR = [

 [ 0x54, 0x65, 0x73, 0x74, 0x20, 0x44, 0x61, 0x74, 0x61 ],

 [ 0xAF, 0xFD, 0x2B, 0x0B, 0xFA, 0x34, 0x33, 0x63, 0x9C ]

]

TV011D\_TV5\_SPLITS : [9][9]GF28

TV011D\_TV5\_SPLITS = [

 [ 0xFB, 0x98, 0x58, 0x7F, 0xDA, 0x70, 0x52, 0x17, 0xFD ],

 [ 0x17, 0x82, 0x25, 0x62, 0xC9, 0x2C, 0x07, 0xB2, 0x44 ],

 [ 0xB8, 0x7F, 0x0E, 0x69, 0x33, 0x18, 0x34, 0xD1, 0xD8 ],

 [ 0xD2, 0xB6, 0xDF, 0x58, 0xEF, 0x94, 0xAD, 0xE5, 0x2B ],

 [ 0x7D, 0x4B, 0xF4, 0x53, 0x15, 0xA0, 0x9E, 0x86, 0xB7 ],

 [ 0x91, 0x51, 0x89, 0x4E, 0x06, 0xFC, 0xCB, 0x23, 0x0E ],

 [ 0x3E, 0xAC, 0xA2, 0x45, 0xFC, 0xC8, 0xF8, 0x40, 0x92 ],

 [ 0x45, 0xDE, 0x36, 0x2C, 0xA3, 0xF9, 0xE4, 0x4B, 0xF5 ],

 [ 0xEA, 0x23, 0x1D, 0x27, 0x59, 0xCD, 0xD7, 0x28, 0x69 ]

]

/\* test split \*/

property TV011D\_TV5\_split\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV5\_P, TV011D\_TV5\_SR) == TV011D\_TV5\_SPLITS ] == ~zero

TV011D\_TV5\_1\_2\_R : [1][2]GF28

TV011D\_TV5\_1\_2\_R = [

 [ GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x01, GF\_ADD(0x01, 0x02)) ]

]

TV011D\_TV5\_8\_9\_R : [1][2]GF28

TV011D\_TV5\_8\_9\_R = [

 [ GF\_DIV(0x09, GF\_ADD(0x08, 0x09)), GF\_DIV(0x08, GF\_ADD(0x08, 0x09)) ]

]

TV011D\_TV5\_1\_2\_SPLITS : [2][9]GF28

TV011D\_TV5\_1\_2\_SPLITS = [

 [ 0xFB, 0x98, 0x58, 0x7F, 0xDA, 0x70, 0x52, 0x17, 0xFD ],

 [ 0x17, 0x82, 0x25, 0x62, 0xC9, 0x2C, 0x07, 0xB2, 0x44 ]

]

TV011D\_TV5\_8\_9\_SPLITS : [2][9]GF28

TV011D\_TV5\_8\_9\_SPLITS = [

 [ 0x45, 0xDE, 0x36, 0x2C, 0xA3, 0xF9, 0xE4, 0x4B, 0xF5 ],

 [ 0xEA, 0x23, 0x1D, 0x27, 0x59, 0xCD, 0xD7, 0x28, 0x69 ]

]

TV011D\_TV5\_SECRET : [1][9]GF28

TV011D\_TV5\_SECRET = [

 [ 0x54, 0x65, 0x73, 0x74, 0x20, 0x44, 0x61, 0x74, 0x61 ]

]

/\* test recombines \*/

property TV011D\_TV5\_recombine\_1\_2\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV5\_1\_2\_R, TV011D\_TV5\_1\_2\_SPLITS) == TV011D\_TV5\_SECRET ] == ~zero

property TV011D\_TV5\_recombine\_8\_9\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV5\_8\_9\_R, TV011D\_TV5\_8\_9\_SPLITS) == TV011D\_TV5\_SECRET ] == ~zero

/\*

 \* Test vector TV011D\_6

 \* secret = 01 02 03 04 05 06 07 08 09 0A 0B 0C 0D 0E 0F

 \* random = 02 4A 89 AC 96 8C 98 65 77 FE B0 24 11 6B 94 F6 54 DD DE 20 9C 3C C3 E4 48 88 4D 31 F8 C8

 \*

 \* l = 15

 \* m = 3

 \* n = 5

 \*

 \* split1 = 49 27 19 F9 8C 92 7D 6A 80 F4 AB 2B CD 72 3F

 \* split2 = 30 87 38 A0 34 EB 94 C2 F2 2B DE 20 87 50 E5

 \* split3 = 78 A2 22 5D BD 7F EE A0 7B D5 7E 07 47 2C D5

 \* split4 = DD 0E 49 40 9F 86 BD B9 15 6F A6 C1 58 10 D4

 \* split5 = 95 2B 53 BD 16 12 C7 DB 9C 91 06 E6 98 6C E4

 \*/

TV011D\_TV6\_P : [5][3]GF28

TV011D\_TV6\_P = [

 [ GF\_POW(0x01, 0x00), GF\_POW(0x01, 0x01), GF\_POW(0x01, 0x02) ],

 [ GF\_POW(0x02, 0x00), GF\_POW(0x02, 0x01), GF\_POW(0x02, 0x02) ],

 [ GF\_POW(0x03, 0x00), GF\_POW(0x03, 0x01), GF\_POW(0x03, 0x02) ],

 [ GF\_POW(0x04, 0x00), GF\_POW(0x04, 0x01), GF\_POW(0x04, 0x02) ],

 [ GF\_POW(0x05, 0x00), GF\_POW(0x05, 0x01), GF\_POW(0x05, 0x02) ]

]

TV011D\_TV6\_SR : [3][15]GF28

TV011D\_TV6\_SR = [

 [ 0x01, 0x02, 0x03, 0x04, 0x05, 0x06, 0x07, 0x08, 0x09, 0x0A, 0x0B, 0x0C, 0x0D, 0x0E, 0x0F ],

 [ 0x02, 0x89, 0x96, 0x98, 0x77, 0xB0, 0x11, 0x94, 0x54, 0xDE, 0x9C, 0xC3, 0x48, 0x4D, 0xF8 ],

 [ 0x4A, 0xAC, 0x8C, 0x65, 0xFE, 0x24, 0x6B, 0xF6, 0xDD, 0x20, 0x3C, 0xE4, 0x88, 0x31, 0xC8 ]

]

TV011D\_TV6\_SPLITS : [5][15]GF28

TV011D\_TV6\_SPLITS = [

 [ 0x49, 0x27, 0x19, 0xF9, 0x8C, 0x92, 0x7D, 0x6A, 0x80, 0xF4, 0xAB, 0x2B, 0xCD, 0x72, 0x3F ],

 [ 0x30, 0x87, 0x38, 0xA0, 0x34, 0xEB, 0x94, 0xC2, 0xF2, 0x2B, 0xDE, 0x20, 0x87, 0x50, 0xE5 ],

 [ 0x78, 0xA2, 0x22, 0x5D, 0xBD, 0x7F, 0xEE, 0xA0, 0x7B, 0xD5, 0x7E, 0x07, 0x47, 0x2C, 0xD5 ],

 [ 0xDD, 0x0E, 0x49, 0x40, 0x9F, 0x86, 0xBD, 0xB9, 0x15, 0x6F, 0xA6, 0xC1, 0x58, 0x10, 0xD4 ],

 [ 0x95, 0x2B, 0x53, 0xBD, 0x16, 0x12, 0xC7, 0xDB, 0x9C, 0x91, 0x06, 0xE6, 0x98, 0x6C, 0xE4 ]

]

/\* test split \*/

property TV011D\_TV6\_split\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV6\_P, TV011D\_TV6\_SR) == TV011D\_TV6\_SPLITS ] == ~zero

TV011D\_TV6\_1\_2\_3\_R : [1][3]GF28

TV011D\_TV6\_1\_2\_3\_R = [

 [ GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x01, 0x03))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x02)), GF\_DIV(0x03, GF\_ADD(0x02, 0x03))],

 GF\_PROD[GF\_DIV(0x01, GF\_ADD(0x01, 0x03)), GF\_DIV(0x02, GF\_ADD(0x02, 0x03))]

 ]

]

TV011D\_TV6\_2\_3\_4\_R : [1][3]GF28

TV011D\_TV6\_2\_3\_4\_R = [

 [ GF\_PROD[GF\_DIV(0x03, GF\_ADD(0x02, 0x03)), GF\_DIV(0x04, GF\_ADD(0x02, 0x04))],

 GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x02, 0x03)), GF\_DIV(0x04, GF\_ADD(0x03, 0x04))],

 GF\_PROD[GF\_DIV(0x02, GF\_ADD(0x02, 0x04)), GF\_DIV(0x03, GF\_ADD(0x03, 0x04))]

 ]

]

TV011D\_TV6\_1\_2\_3\_SPLITS : [3][15]GF28

TV011D\_TV6\_1\_2\_3\_SPLITS = [

 [ 0x49, 0x27, 0x19, 0xF9, 0x8C, 0x92, 0x7D, 0x6A, 0x80, 0xF4, 0xAB, 0x2B, 0xCD, 0x72, 0x3F ],

 [ 0x30, 0x87, 0x38, 0xA0, 0x34, 0xEB, 0x94, 0xC2, 0xF2, 0x2B, 0xDE, 0x20, 0x87, 0x50, 0xE5 ],

 [ 0x78, 0xA2, 0x22, 0x5D, 0xBD, 0x7F, 0xEE, 0xA0, 0x7B, 0xD5, 0x7E, 0x07, 0x47, 0x2C, 0xD5 ]

]

TV011D\_TV6\_2\_3\_4\_SPLITS : [3][15]GF28

TV011D\_TV6\_2\_3\_4\_SPLITS = [

 [ 0x30, 0x87, 0x38, 0xA0, 0x34, 0xEB, 0x94, 0xC2, 0xF2, 0x2B, 0xDE, 0x20, 0x87, 0x50, 0xE5 ],

 [ 0x78, 0xA2, 0x22, 0x5D, 0xBD, 0x7F, 0xEE, 0xA0, 0x7B, 0xD5, 0x7E, 0x07, 0x47, 0x2C, 0xD5 ],

 [ 0xDD, 0x0E, 0x49, 0x40, 0x9F, 0x86, 0xBD, 0xB9, 0x15, 0x6F, 0xA6, 0xC1, 0x58, 0x10, 0xD4 ]

]

TV011D\_TV6\_SECRET : [1][15]GF28

TV011D\_TV6\_SECRET = [

 [ 0x01, 0x02, 0x03, 0x04, 0x05, 0x06, 0x07, 0x08, 0x09, 0x0A, 0x0B, 0x0C, 0x0D, 0x0E, 0x0F ]

]

/\* test recombines \*/

property TV011D\_TV6\_recombine\_1\_2\_3\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV6\_1\_2\_3\_R, TV011D\_TV6\_1\_2\_3\_SPLITS) == TV011D\_TV6\_SECRET ] == ~zero

property TV011D\_TV6\_recombine\_2\_3\_4\_passes\_test = [ GF\_MAT\_MUL(TV011D\_TV6\_2\_3\_4\_R, TV011D\_TV6\_2\_3\_4\_SPLITS) == TV011D\_TV6\_SECRET ] == ~zero

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